# ECE 2025 Homework 8 Solutions



8.1 (a) $y[n] = \frac{1}{2} \sum_{k=1}^{5} x[n-k]$

$$h[n] = \frac{1}{2} \sum_{k=1}^{5} \delta[n-k]$$

$$H(e^{j\hat{\omega}}) = \frac{1}{2} \sum_{k=1}^{5} e^{-j\hat{\omega}k} = e^{-j\hat{\omega}3}\left(\frac{1}{2} + \cos\hat{\omega} + \cos 2\hat{\omega}\right)$$

$$H(z) = \frac{1}{2} \sum_{k=1}^{5} z^{-k} = \frac{1}{2} \frac{z^{-1} - z^{-6}}{1 - z^{-1}}$$

(b) $h[n] = u[n-5] - u[n-2]$

$$= -\delta[n-2] - \delta[n-3] - \delta[n-4]$$

$$y[n] = -x[n-2] - x[n-3] - x[n-4]$$

$$H(e^{+j\hat{\omega}}) = -\sum_{k=2}^{4} e^{-j\hat{\omega}k} = -2e^{-j\hat{\omega}3}\left(\frac{1}{2} + \cos\hat{\omega}\right)$$

$$H(z) = -\sum_{k=2}^{4} z^{-k} = -z^{-2} - z^{-3} - z^{-4}$$

(c) $H(e^{j\hat{\omega}}) = (2 + \cos 3\hat{\omega})\, e^{-j4\hat{\omega}} = \left(2 + \frac{e^{j\hat{\omega}3} + e^{-j\hat{\omega}3}}{2}\right) e^{-j\hat{\omega}4}$

$$= \frac{1}{2} e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}4} + \frac{1}{2} e^{-j\hat{\omega}7}$$

$$H(z) = \frac{1}{2} z^{-1} + 2z^{-4} + \frac{1}{2} z^{-7}$$

$$y[n] = \frac{1}{2} x[n-1] + 2x[n-4] + \frac{1}{2} x[n-7]$$

$$h[n] = \frac{1}{2}\delta[n-1] + 2\delta[n-4] + \frac{1}{2}\delta[n-7]$$

8.2 (a) $H(z) = z^{-1}$

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}$$

$$y[n] = x[n-1]$$

$$h[n] = \delta[n-1]$$

(b) $H(z) = 2 + 3z^{-4} - z^{-8}$

$$H(e^{j\hat{\omega}}) = 2 + 3e^{-j\hat{\omega}4} - e^{-j\hat{\omega}8}$$

$$y[n] = 2x[n] + 3x[n-4] - x[n-8]$$

$$h[n] = 2\delta[n] + 3\delta[n-4] - \delta[n-8]$$

(c) $H(z) = \dfrac{1-z^{-6}}{1-z^{-1}} = \displaystyle\sum_{k=0}^{5} z^{-k}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{5} e^{-j\hat{\omega}k} = \frac{\sin(\hat{\omega}3)}{\sin(\hat{\omega}/2)} e^{-j\hat{\omega}5/2}$$

$$y[n] = \sum_{k=0}^{5} x[n-k]$$

$$h[n] = \sum_{k=0}^{5} \delta[n-k]$$

(d) $H(z) = (1+z^{-1})(1-\sqrt{2}e^{j\pi/4}z^{-1})(1-\sqrt{2}e^{-j\pi/4}z^{-1})$

$$= (1+z^{-1})(1-2z^{-1}+2z^{-2}) = 1 - z^{-1} + 2z^{-3}$$

$$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}3}$$

$$y[n] = x[n] - x[n-1] + 2x[n-3]$$

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$$

8.4 $\quad x[n] = x(n/f_s) = 8\cos(\frac{5}{4}\pi n - \frac{5\pi}{2}) + 7\cos(\frac{14}{5}\pi n)$

$$y[n] = x[n] - x[n-5]$$

$$= 8\cos(\tfrac{5}{4}\pi n - \tfrac{5\pi}{2}) + 7\cos(\tfrac{14}{5}\pi n)$$

$$- 8\cos(\tfrac{5}{4}\pi n - \tfrac{35}{4}\pi) - 7\cos(\tfrac{14}{5}\pi n - 14\pi)$$

$$= 8\left(\cos(\tfrac{5}{4}\pi n - \tfrac{\pi}{2}) - \cos(\tfrac{5}{4}\pi n - \tfrac{3\pi}{4})\right)$$

$$= 8\left(\cos(\tfrac{3}{4}\pi n + \tfrac{\pi}{2}) - \cos(\tfrac{3}{4}\pi n + \tfrac{3\pi}{4})\right)$$

$$y(t) = y[f_s t] = 8\left(\cos(6000\pi t + \tfrac{\pi}{2}) - \cos(6000\pi t + \tfrac{3}{4}\pi)\right)$$

8.5 (a) $\quad H(z) = 1 - z^{-5}$

(b) $\quad H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}5}$

(c) $\quad y[n] = 8\left(\cos(\frac{5\pi}{2}n - \frac{5\pi}{2}) - \cos(\frac{5\pi}{2}n - 15\pi)\right)$

$$= 8\left(\cos(\tfrac{\pi}{2}n - \tfrac{\pi}{2}) + \cos(\tfrac{\pi}{2}n)\right)$$

$$y(t) = y[f_s t] = 8\left(\cos(\omega t - \tfrac{\pi}{2}) + \cos(\omega t)\right)$$

where $\omega = \frac{\pi}{2} f_s = \frac{\pi}{2} \times 11025$

8.6 (a) $\left(\delta(t-2)+u(t+2)\right) * \delta(t-3)$

$$= \delta(t-5) + u(t-1)$$

(b) $\left(e^{-4t}u(t-0.5) + 3\sin(5\pi t)\,u(t)\right)\delta(t-0.1)$

$$= \left(e^{-4t}u(t-0.5) + 3\sin(5\pi t)\,u(t)\right)\Big|_{t=0.1}\,\delta(t-0.1)$$

$$= 3\sin(0.5\pi)\,\delta(t-0.1) = 3\,\delta(t-0.1)$$

(c) $\int_{-\infty}^{t-3}\delta(\tau-2)\,e^{-\pi\tau}u(\tau)\,d\tau$

$$= \int_{-\infty}^{t-3}\delta(\tau-2)\,e^{-\pi 2}u(2)\,d\tau$$

$$= e^{-\pi 2}\int_{-\infty}^{t-3}\delta(\tau-2)\,d\tau = e^{-\pi 2}\,u(t-5)$$

(d) $\frac{d}{dt}\left(\cos(5t)\left(u(t)-u(t-4)\right)\right)$

$$= -5\sin(5t)\left(u(t)-u(t-4)\right) + \cos(5t)\left(\delta(t)-\delta(t-4)\right)$$

$$= -5\sin(5t)\left(u(t)-u(t-4)\right) + \delta(t) - \cos(20)\,\delta(t-4)$$