PROBLEM 7.1*:

In the *cascade connection* of Fig. 1, Systems #1 and #2 are described by a difference equation and an impulse response:

$$y_1[n] = \tfrac{1}{2}x[n] + x[n-1] + \tfrac{1}{2}x[n-2] \qquad \text{and} \qquad h_2[n] = \tfrac{1}{2}\delta[n-1] + \delta[n-2] + \tfrac{1}{2}\delta[n-3]$$

Notice that the two filters are nearly identical: Filter #2 is a delayed version of Filter #1.

(a) The frequency response sequence, $H_1(e^{j\hat{\omega}})$, of the first system is:

$$H_1(e^{j\hat{\omega}}) = \tfrac{1}{2} + e^{-j\hat{\omega}} + \tfrac{1}{2}e^{-j2\hat{\omega}} = e^{-j\hat{\omega}}\left(\tfrac{1}{2}e^{j\hat{\omega}} + 1 + \tfrac{1}{2}e^{-j\hat{\omega}}\right) = e^{-j\hat{\omega}}\left(1 + \cos(\hat{\omega})\right)$$

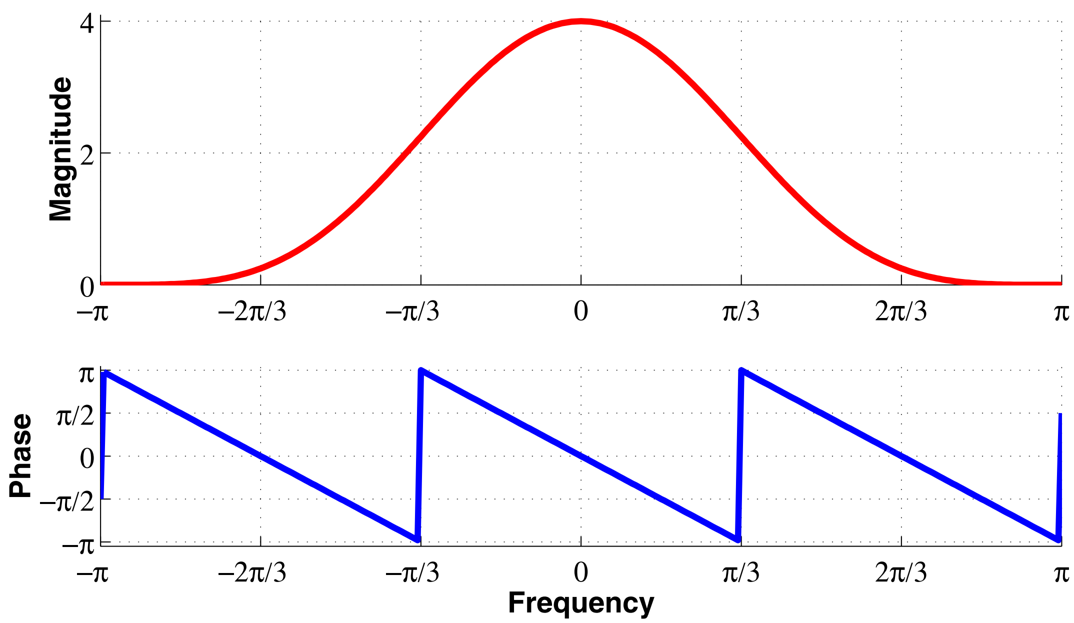
(b) The frequency response sequence, $H_2(e^{j\hat{\omega}})$, of the second system is:

$$H_2(e^{j\hat{\omega}}) = \tfrac{1}{2}e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + \tfrac{1}{2}e^{-j2\hat{\omega}} = e^{-j2\hat{\omega}}\left(\tfrac{1}{2}e^{j\hat{\omega}} + 1 + \tfrac{1}{2}e^{-j\hat{\omega}}\right) = e^{-j2\hat{\omega}}\left(1 + \cos(\hat{\omega})\right)$$

Thus the frequency response, $H(e^{j\hat{\omega}})$, of the overall cascade system is:

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}\left(1 + \cos(\hat{\omega})\right)e^{-j2\hat{\omega}}\left(1 + \cos(\hat{\omega})\right) = e^{-j3\hat{\omega}}\left(1 + \cos(\hat{\omega})\right)^2$$

(c) The plot of the magnitude and phase of the frequency response of the overall cascaded system is below:



(d) When the input to this system is

$$x[n] = 5 + 3\cos(\tfrac{1}{3}\pi(n-1))$$

we evaluate the frequency response at $\hat{\omega} = 0, \tfrac{1}{3}\pi$ to determine the output signal, $y[n]$.

$$\text{at } \hat{\omega} = 0, \quad H(e^{j\hat{\omega}})\big|_{\hat{\omega}=0} = e^{-j3\hat{\omega}}\left(1 + \cos(\hat{\omega})\right)^2 = 2$$

$$\text{at } \hat{\omega} = \tfrac{1}{3}\pi, \quad H(e^{j\hat{\omega}})\big|_{\hat{\omega}=\pi/3} = e^{-j3\pi/3}\left(1 + \cos(\pi/3)\right)^2 = e^{-j\pi}\left(1 + \tfrac{1}{2}\right)^2 = 2.25e^{-j\pi} = -2.25$$

Thus the output signal is:

$$y[n] = 5 \times 4 + (3 \times 2.25)\cos(\tfrac{1}{3}\pi(n-1) - \pi) = 20 + 6.75\cos(\tfrac{1}{3}\pi n - 4\pi/3)$$

This formula for $y[n]$ is valid over the range $-\infty \leq n \leq \infty$.

PROBLEM 7.2*:

7.2(a) $$\boxed{H(e^{j\hat{w}}) = 1 - e^{-j\hat{w}} + e^{-j2\hat{w}} - e^{-j3\hat{w}} + e^{-j4\hat{w}} - e^{-j5\hat{w}}}$$

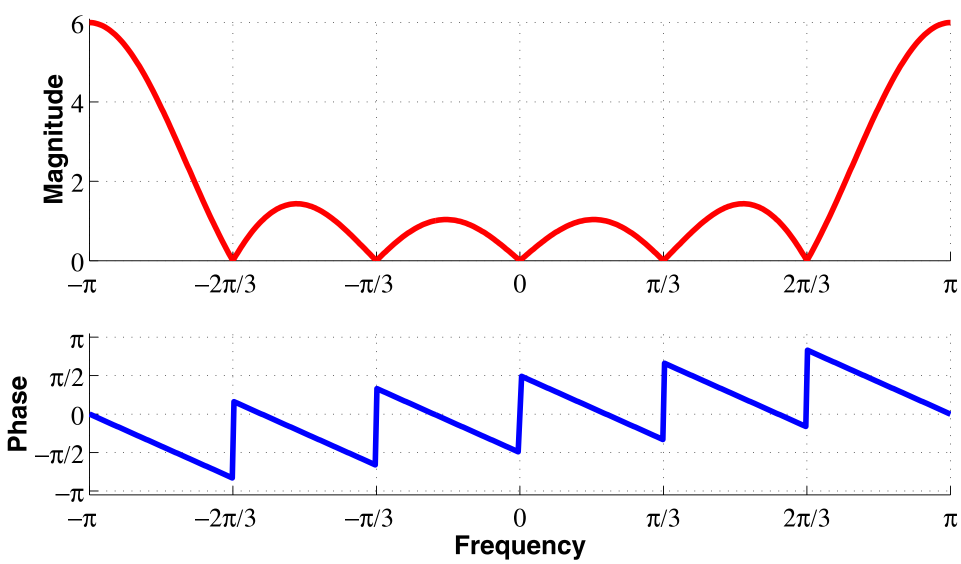
7.2(b) Refer to page 145 in the text.
Rewrite $H(e^{j\hat{w}})$ as

$$H(e^{jw}) = 1 + e^{-j\hat{w}}e^{-j\pi} + e^{-j2\hat{w}} \cdot e^{-j2\pi} + e^{-j3\hat{w}}e^{-j3\pi} + e^{-j4\hat{w}}e^{-j4\pi} + e^{-j5\hat{w}}e^{-j5\pi}$$

$$H(e^{jw}) = 1 + e^{-j(\hat{w}-\pi)} + e^{-j2(\hat{w}-\pi)} + e^{-j3(\hat{w}-\pi)} + e^{-j4(\hat{w}-\pi)} + e^{-j5(\hat{w}-\pi)}$$

then using equation 6.25 on page 145:

$$L=6, \quad H(e^{j\hat{w}}) = \frac{\sin\left(3(\hat{w}-\pi)\right)}{\sin\left(\frac{1}{2}(\hat{w}-\pi)\right)} e^{-j2.5(\hat{w}-\pi)}$$



7.2(d) $\sin(3(\hat{w}-\pi)) = 0$ at $\boxed{\hat{w} = 0, \pm\frac{\pi}{3}, \pm\frac{2\pi}{3} = \pm 1.0472, \pm 2.0944}$

PROBLEM 7.3*:

7.3 $y[n] = \frac{1}{2}\delta[n-4] + \delta[n-5] + \frac{1}{2}\delta[n-6]$

7.3(a) $n = -\infty$ to ∞

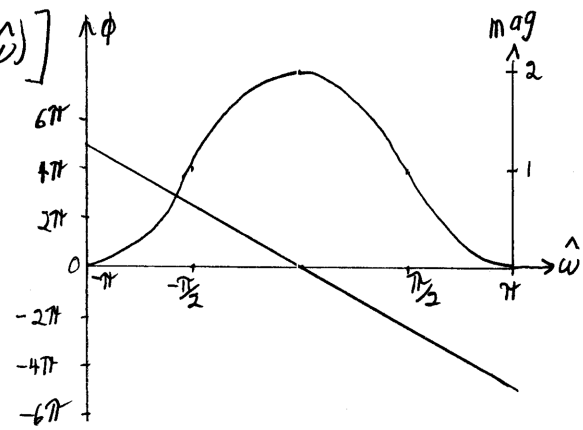
$h[n] =$					$1/2$	1	$1/2$			
$x[n] =$	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2
	$C_1/2$	$C_2/2$	$C_1/2$	$C_2/2$	$C_1/2$	$C_2/2$	$C_1/2$	$C_2/2$	$C_1/2$	$C_2/2$
	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	$C_1 \cdots$
	$C_1/2$	$C_2/2$	$C_1/2$	$C_2/2$	$C_1/2$	$C_2/2$	$C_1/2$	$C_2/2$	$C_1/2$	$C_2/2$
$y[n] =$	(C_1+C_2)	(C_1+C_2)	(C_1+C_2)	(C_1+C_2)	(C_1+C_2)	(C_1+C_2)	(C_1+C_2)	(C_1+C_2)	$\cdots$	

7.3(b) $H(e^{j\hat{\omega}}) = \frac{1}{2}e^{-j4\hat{\omega}} + e^{-j5\hat{\omega}} + \frac{1}{2}e^{-j6\hat{\omega}}$

$= e^{-j5\hat{\omega}}\left[\frac{1}{2}e^{+j\hat{\omega}} + 1 + \frac{1}{2}e^{-j\hat{\omega}}\right]$

$= e^{-j5\hat{\omega}}\left[1 + \cos(\hat{\omega})\right]$

7.3(c)



7.3(d) $x_1[n] = 3(-1)^n + 2\cos(0.625\pi n)$

There are two components in $x_1[n]$. The first component, $3(-1)^n$ can be shown to produce a zero output, in accordance with 7.3(a). For the second component

$H(\hat{\omega} = 0.625\pi) = e^{-j5(0.625\pi)}\left[1 + \cos(0.625\pi)\right]$

$= 0.6173\,e^{-j\frac{9\pi}{8}} = 0.6173\,e^{j\frac{7\pi}{8}}$

$x_1[n] = 2(0.6173)\cos\left[0.625\pi n + \frac{7\pi}{8}\right]$

7.4(a) $H(e^{j\hat{\omega}}) = \frac{1}{2}e^{-j4\hat{\omega}} + e^{-j5\hat{\omega}} + \frac{1}{2}e^{-j6\hat{\omega}}$

$= e^{-j5\hat{\omega}}\left(1 + \frac{1}{2}e^{+j\hat{\omega}} + \frac{1}{2}e^{-j\hat{\omega}}\right) = e^{-j5\hat{\omega}}(1+\cos\hat{\omega})$

7.4(b)

$x(t) = \cos\omega t \qquad y(t) = \cos(\omega t + \phi)$

The magnitude of $H(e^{j\hat{\omega}})$ must be $=1$, so that requires $\cos\hat{\omega} = 0$, and $\therefore\ \hat{\omega} = \pi/2 = 2\pi \frac{f}{f_s}$

$\therefore f = \frac{\pi}{2} f_s \cdot \frac{1}{2\pi} = \frac{1}{4} f_s = \frac{800}{4} = 200\,Hz.$

$x(t) = \cos[400\pi t]$

7.4(c) $x(t) = 99 + 88\cos(500\pi t)$

for $f_s = 800$

$x[n] = 99 + 88\cos\left(\frac{5\pi n}{8}\right)$

$H\left(\hat{\omega} = \frac{5\pi}{8}\right) = e^{-j\frac{25\pi}{8}}\left(1 + \cos\frac{5\pi}{8}\right) = 0.617e^{-j\frac{25\pi}{8}}$

$H(\hat{\omega} = 0) = e^{-j0}(1+1) = 2$

$x(t) = 99(2) + 88(.617)\cos(500\pi t - 25\pi/8)$

$= 198 + 54.32\cos(500\pi t - 25\pi/8)$

PROBLEM 7.5*:

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

$$\begin{aligned}
H(e^{j\hat{\omega}}) &= \left(1+e^{-j2\hat{\omega}}\right)\left(1+e^{-j4\pi/3}e^{-j\hat{\omega}}\right)\left(1+e^{-j2\pi/3}e^{-j\hat{\omega}}\right)\\
&= \left(1+e^{-j2\hat{\omega}}\right)\left(1+\left(e^{+j2\pi/3}+e^{-j2\pi/3}\right)e^{-j\hat{\omega}}+e^{-j2\hat{\omega}}\right)\\
&= 1+2\cos(2\pi/3)e^{-j\hat{\omega}}+(1+1)e^{-j2\hat{\omega}}+2\cos(2\pi/3)e^{-j3\hat{\omega}}+e^{-j4\hat{\omega}}\\
&= 1-e^{-j\hat{\omega}}+2e^{-j2\hat{\omega}}-e^{-j3\hat{\omega}}+e^{-j4\hat{\omega}}
\end{aligned}$$

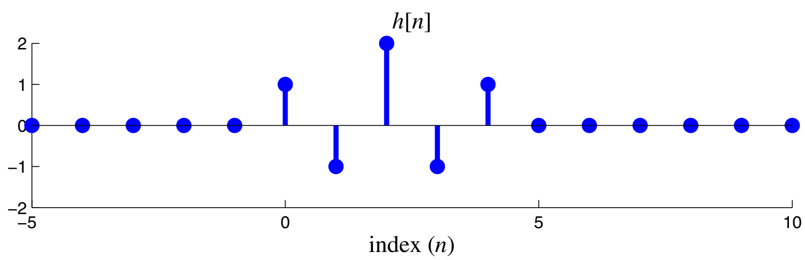
The difference equation is

$$y[n] = x[n] - x[n-1] + 2x[n-2] - x[n-3] + x[n-4]$$

(b) Determine the impulse response of this system by using the coefficients of the expression for $H(e^{j\hat{\omega}})$.

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$

Stem Plot is here



(c) If the input is a complex exponential of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, then $y[n] = 0$ for all n, whenever $H(e^{j\hat{\omega}}) = 0$ for that $\hat{\omega}$. Using the factored form of $H(e^{j\hat{\omega}})$, we can set each factor equal to zero:

$$\begin{aligned}
\left(1+e^{-j2\hat{\omega}}\right) = 0 &\quad\Longrightarrow \hat{\omega} = \pm\pi/2\\
\left(1+e^{-j4\pi/3}e^{-j\hat{\omega}}\right) = 0 &\quad\Longrightarrow \hat{\omega} = \pi - 4\pi/3 = -\pi/3\\
\left(1+e^{-j2\pi/3}e^{-j\hat{\omega}}\right) = 0 &\quad\Longrightarrow \hat{\omega} = \pi - 2\pi/3 = \pi/3
\end{aligned}$$

(d) The output of this system when the input is

$$x[n] = 3 + 7\delta[n-1] + 13\cos(0.5\pi n - \pi/4) \qquad \text{for } -\infty < n < \infty$$

can be determined by *Superposition*.

$$x_1[n] = 3 \implies y_1[n] = H(e^{j0}) \times 3 = 2 \times 3 = 6$$

$$\begin{aligned}
x_2[n] = 7\delta[n-1] \implies y_2[n] &= 7h[n-1]\\
&= 7\delta[n-1] - 7\delta[n-2] + 14\delta[n-3] - 7\delta[n-4] + 7\delta[n-5]
\end{aligned}$$

$$x_3[n] = 13\cos(0.5\pi n - \pi/4) \implies y_3[n] = 13\left|H(e^{j0.5\pi})\right|\cos\left(0.5\pi n - \pi/4 + \angle H(e^{j0.5\pi})\right) = 0$$

because we have already noted that $H(e^{j\hat{\omega}}) = 0$ at $\hat{\omega} = \pm\pi/2$. Finally, we get

$$\begin{aligned}
y[n] &= y_1[n] + y_2[n] + y_3[n]\\
&= 6 + 7\delta[n-1] - 7\delta[n-2] + 14\delta[n-3] - 7\delta[n-4] + 7\delta[n-5]
\end{aligned}$$