5.1(a) $x[n] = 7\cos(0.2\pi n + \pi/4)$

$f_s = 8000$ $(\hat{\omega} = 2\pi)$ $f_{max} = 4000$ $(\hat{\omega} = \pi)$

$f_x = 0.2(4000) = 800\ Hz$

Possible input frequencies $= n(8000) \pm f_x$

n	$n(8000) \pm 800$	
0	$\pm$ 800	
1	7,200,	8,800
2	15,200,	16,800
3	23,200,	24,800
4	31,200,	32,800
5	39,200,	$\boxed{40,800}$
6	$\boxed{47,200}$	48,800

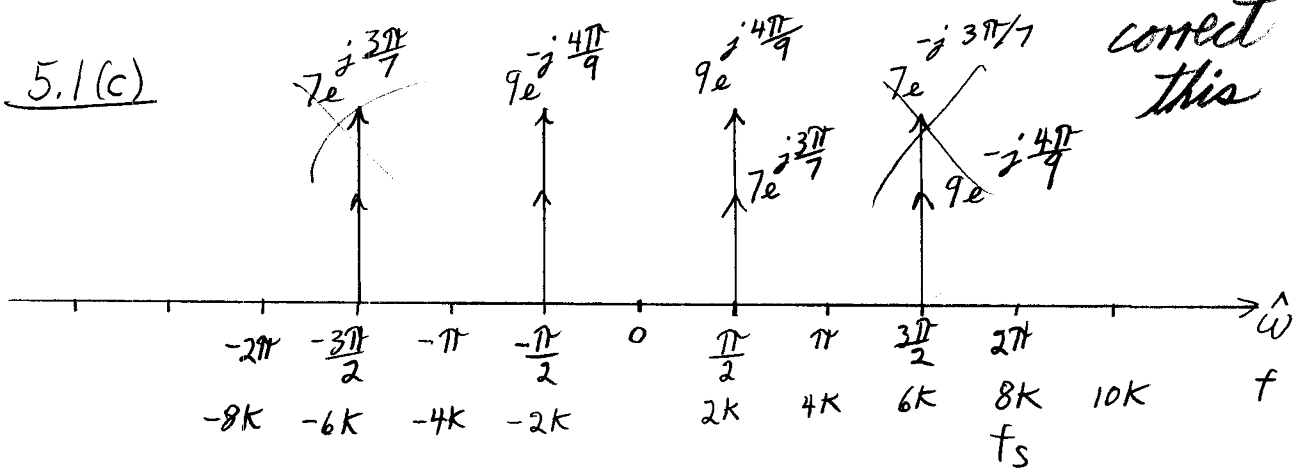
$\boxed{x_1(t) = 7\cos(2\pi \cdot 40{,}800 + \pi/4)}$

$\boxed{x_2(t) = 7\cos(2\pi \cdot 47{,}200 - \pi/4)}$

$x_2(t)$ has negative phase because it is an alias of $\hat{\omega} = -0.2\pi$

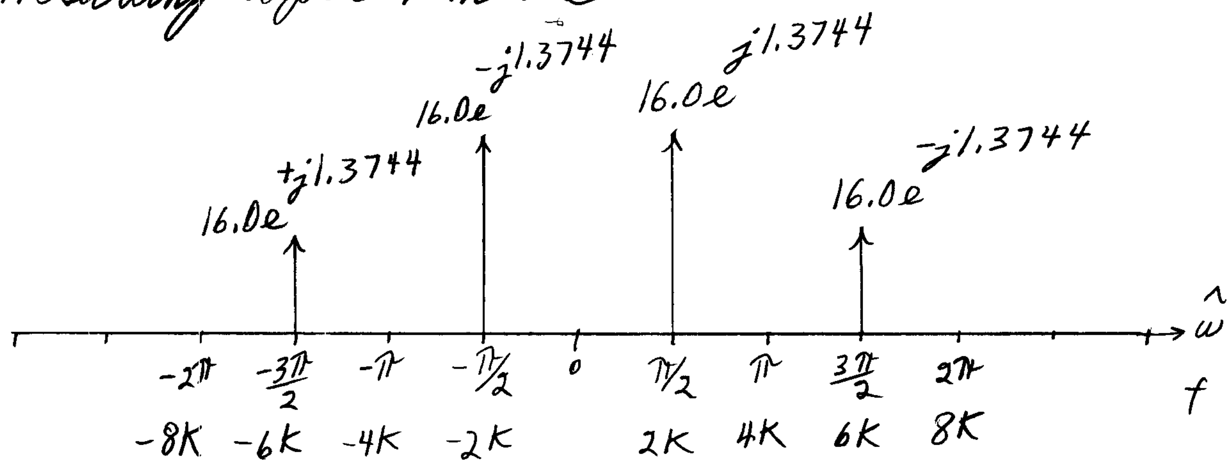
5.1(b) $f_s(\text{minimum}) = 12{,}000$

5.1(c)

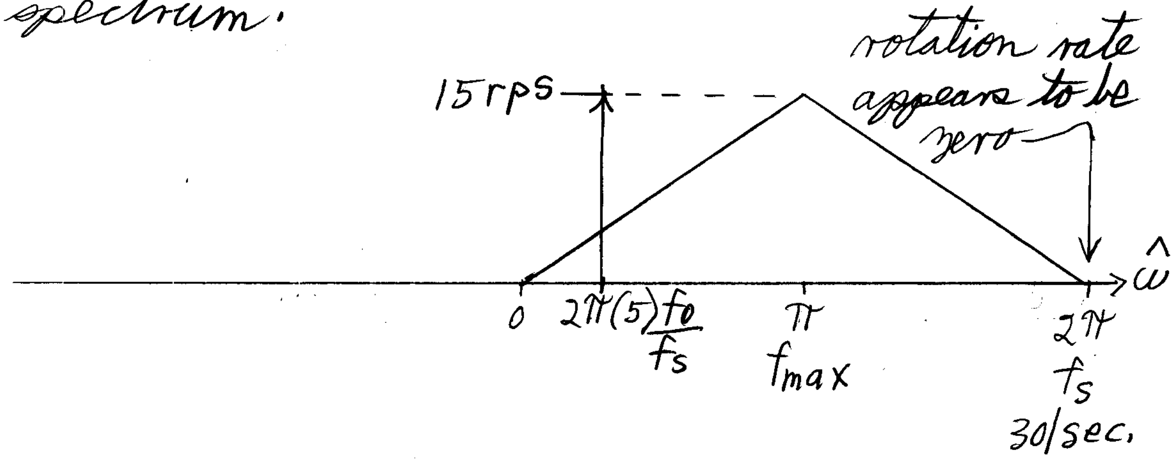


correct this

The double spectral lines are a result of aliasing. When the two components are added together at each frequency, the resulting spectrum is:



5.2 Model the rotating wheel by $e^{j\omega_0 t}$ for $\omega_0 > 0$. Because of the 5 spokes, the signal looks like $e^{j5\omega_0 t} = e^{j5(2\pi f_0 t)}$, since all the spokes look the same. Consider the spectrum:



For $\hat{\omega} = 2\pi(5)\frac{f_0}{f_s} = k2\pi$, the wheel appears stationary. Simplifying, $f_s = \frac{5f_0}{k}$, $k = 1, 2, 3 \cdots$,

Now, if the wheel appears to rotate CCW, $\hat{\omega} < 2\pi$, and $\hat{\omega} = k2\pi - \frac{2\pi}{15}$.

$$\hat{\omega} = 2\pi(5)\frac{f_0}{f_s} = k2\pi - \frac{2\pi}{15}$$

$$f_0 = \frac{f_s}{5}\left(k - \frac{1}{15}\right) = 6k - \frac{6}{15} \quad (\text{for } f_s = 30)$$

The relation between speed S and f_0 is $S = f_0 \frac{\text{rev}}{\text{sec.}} \times \pi(.6\,\text{m}) \times \frac{1\,\text{km}}{10^3\,\text{m}} \times \frac{3600\,\text{sec}}{1\,\text{hr}}$

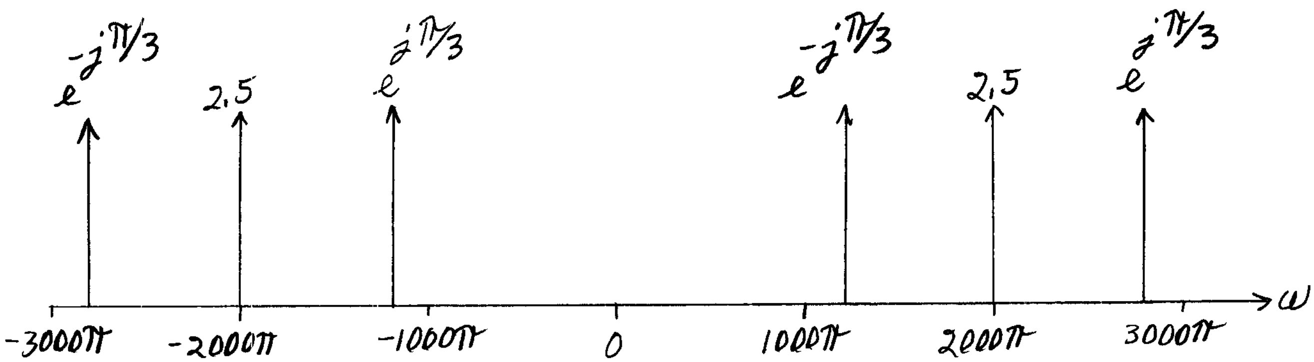
$$S\left(\frac{\text{km}}{\text{hr}}\right) = 6.79\, f_0\left(\frac{\text{rev}}{\text{sec}}\right)$$

$$= 6.79\left(6k - \frac{6}{15}\right) \quad (\text{for } f_s = 30)$$

5.3 $x(t) = [5 + 4\cos(800\pi t + \pi/3)]\cos(2000\pi t)$

(a) $$x(t) = \left[5 + 2e^{j(800\pi t + \pi/3)} + 2e^{-j(800\pi t + \pi/3)}\right] \times \left[\frac{1}{2}e^{j2000\pi t} + \frac{1}{2}e^{-j2000\pi t}\right]$$

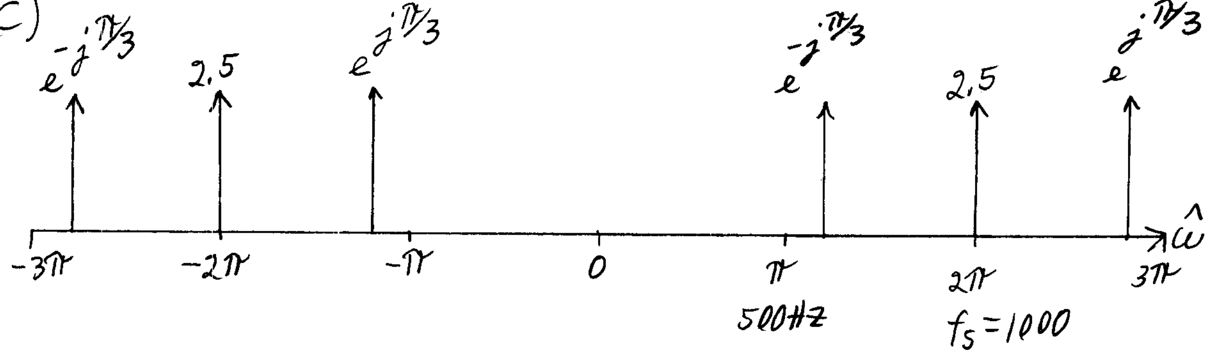
$$x(t) = 2.5e^{j2000\pi t} + 2.5e^{-j2000\pi t} + e^{j(2800\pi t + \pi/3)} + e^{-j(1200\pi t - \pi/3)} + e^{j(1200\pi t - \pi/3)} + e^{-j(2800\pi t + \pi/3)}$$



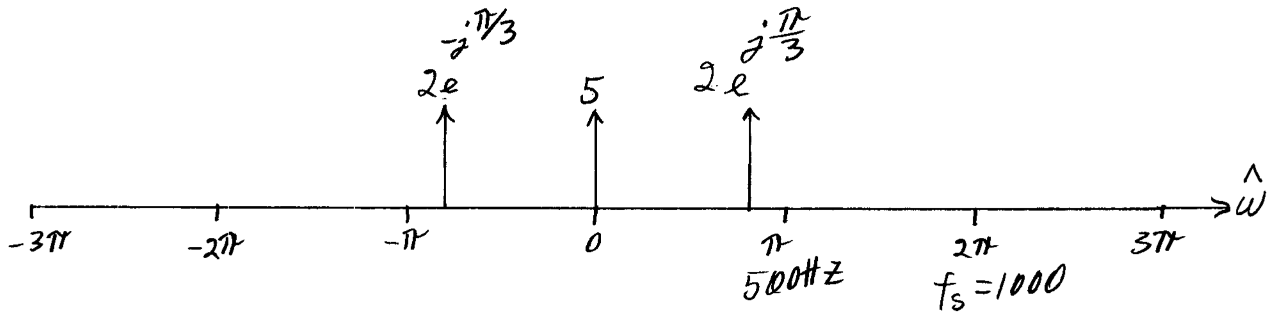
The GCD is $\omega = 400\pi$ $(f = 200)$, $\therefore T = \frac{1}{200} = \boxed{0.05\text{ sec.}}$
The waveform *is* *periodic*.

(b) $f_{s_{min}} = 2\frac{(2800\pi)}{2\pi} = \boxed{2800\text{ samples/sec.}}$

(c)



Because of aliasing, this becomes:



(d) $x(t) = 5 + 4\cos(800\pi t + \pi/3)$

5.4(a) The MATLAB code generates the signal

$$x(t) = \cos\left[5000\pi t + 200\cos(6\pi t)\right]$$

The instantaneous frequency is

$$\omega_i = 5000\pi - 1200\pi \sin(6\pi t)$$

or

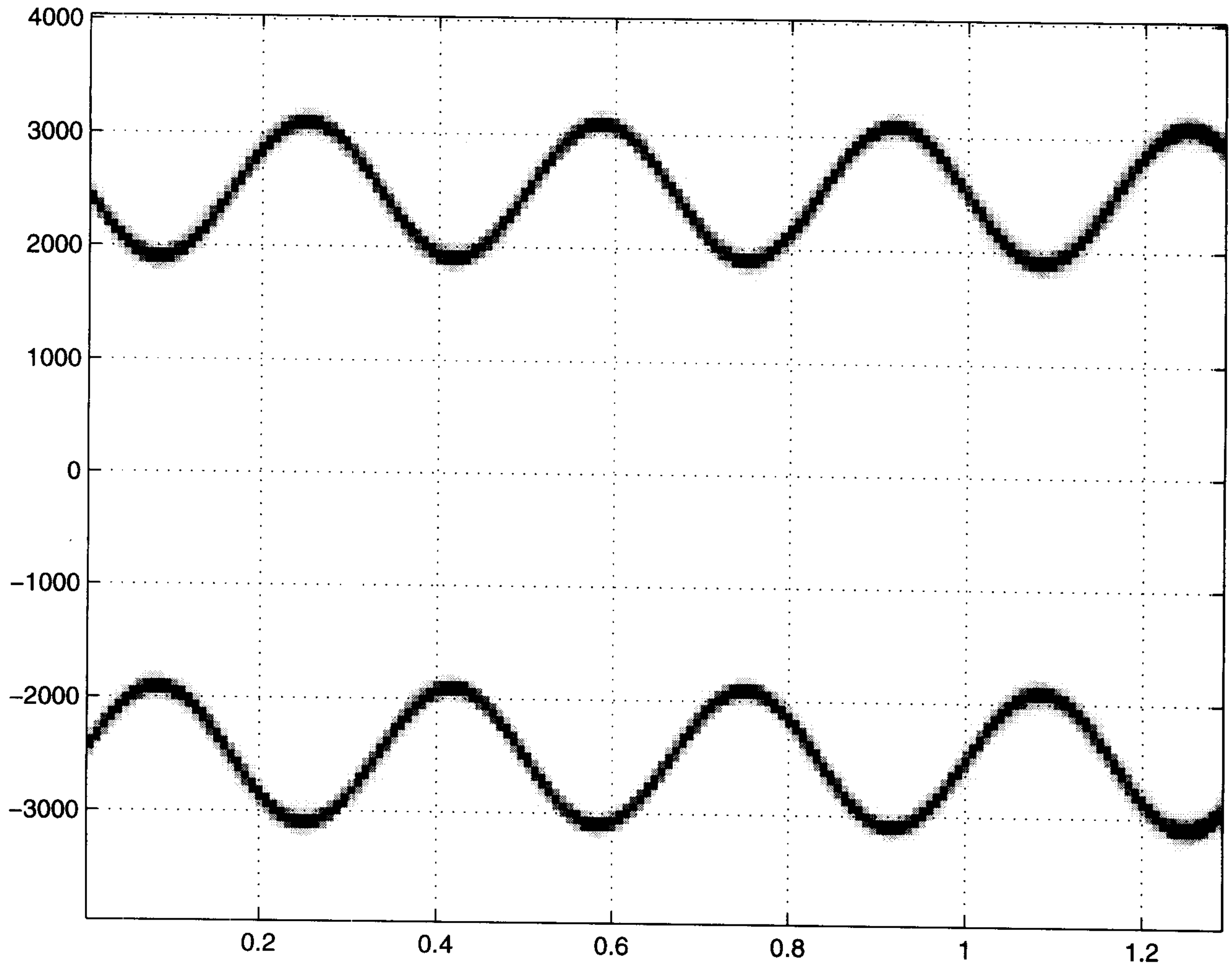
$$f_i = 2500 \mp 600 \sin(6\pi t)$$

Since f_i is sometimes $> f_s/2$, there will be aliasing. ($f_s = 5000$)

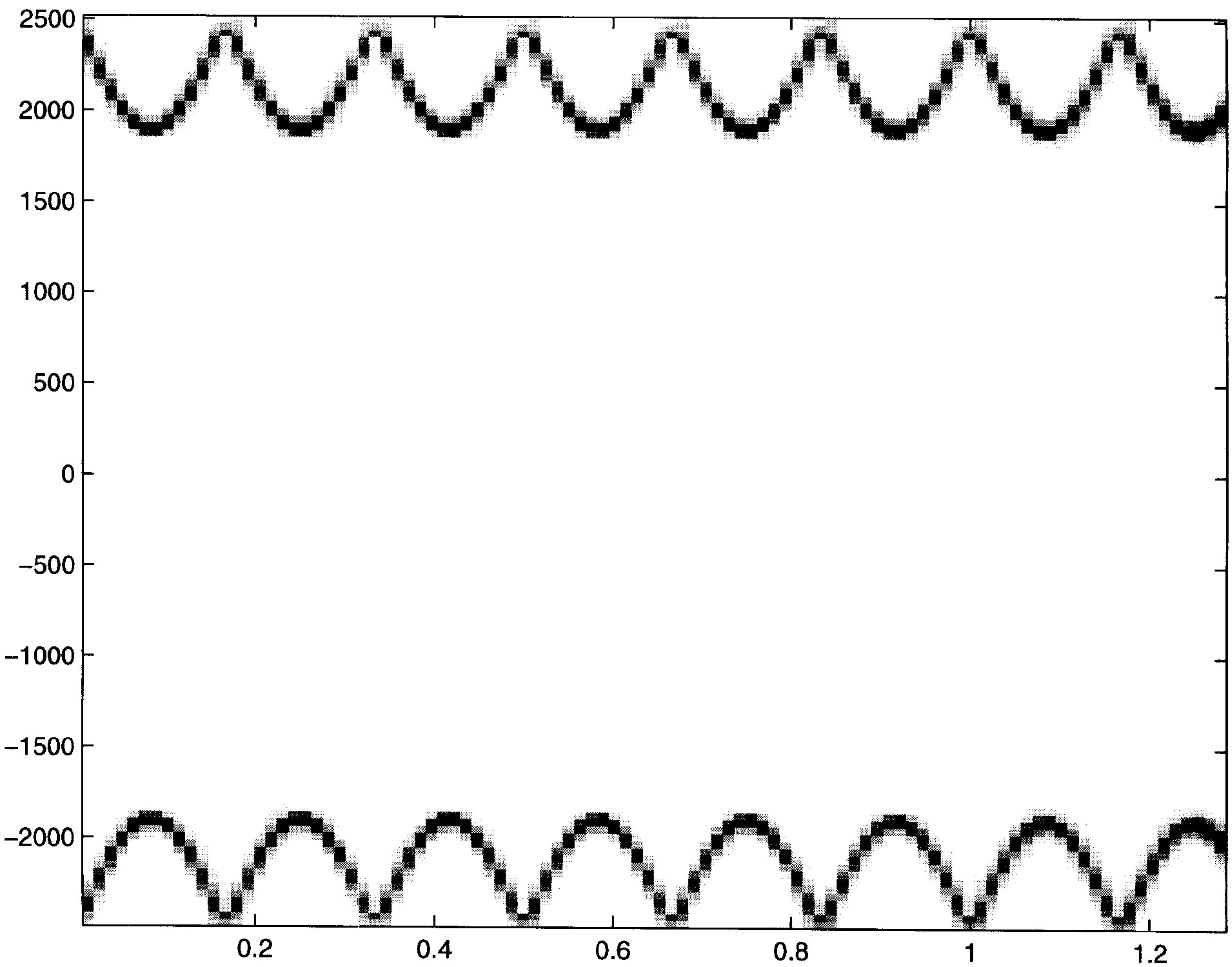
5.4(a) (continued)

$$x[n] = \cos\left[\frac{5000\pi}{f_s} n + 200\cos\left(\frac{6\pi}{f_s} n\right)\right]$$

(b)



5.4(c)

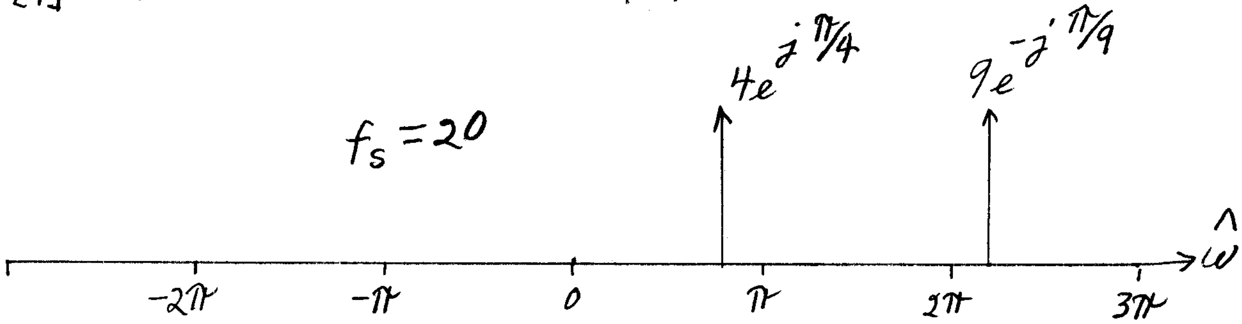


5.5(a) $x(t) = 9e^{j(44\pi t - \pi/9)} + 4e^{j(16\pi t + \frac{\pi}{4})}$

$x[n] = 9e^{j(\frac{44}{20}\pi n - \pi/9)} + 4e^{j(\frac{16}{20}\pi n + \pi/4)}$

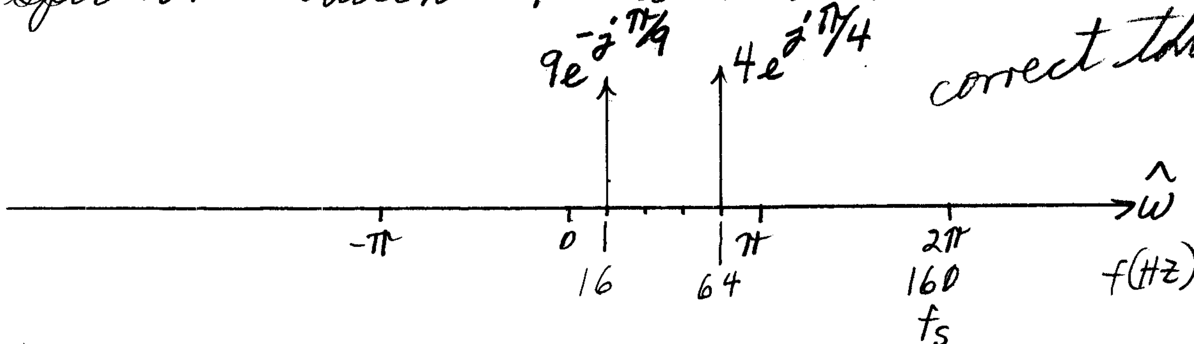
$x[n] = 9e^{j(2.2\pi n - \pi/9)} + 4e^{j(0.8\pi n + \pi/4)}$

$f_s = 20$



but, when aliasing is considered, the spectrum between $-\pi$ and π becomes:

correct this



(b) From the spectrum plot, it is obvious that $f_s = 160$ Hz and so the continuous time spectrum is