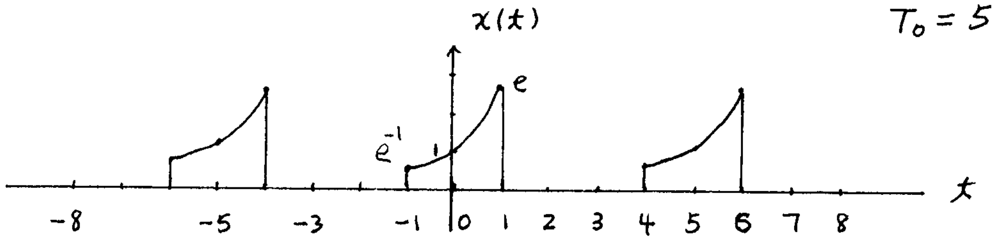
4.1 (a)



(b) $$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{5}\int_{-1}^{1} e^{t}\,dt = \frac{1}{5}(e - e^{-1})$$

(c) $$a_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j(2\pi/T_0)kt}\,dt$$

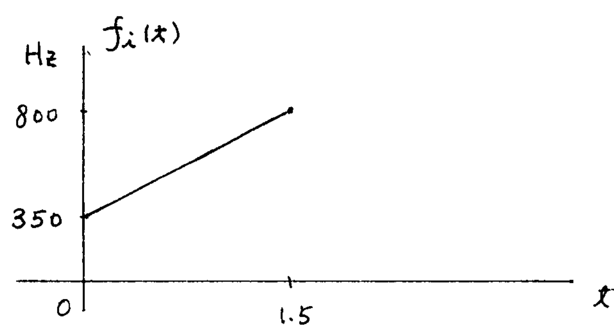
$$= \frac{1}{5}\int_{-1}^{1} e^{t}\, e^{-j(2\pi/5)kt}\,dt$$

4.2 (a) The angle function is

$$\psi(t) = 300\pi t^2 + 700\pi t + 0.3\pi$$

so the instantaneous frequency is

$$f_i(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt} = 300t + 350$$



4.2 (b) The instantaneous frequency is

$$f_i(t) = -1000\,t + 2000$$

so

$$\psi(t) = -2\pi \times 500\,t^2 + 2\pi \times 2000\,t + \phi$$

with ϕ arbitrary.

```
tt = 0:0.0001:1.8;
aa = -1000*pi;
bb = 4000*pi;
cc = 0.3*pi;    // cc is arbitrary
psi = aa*tt.*tt + bb*tt + cc;
xx = real(exp(j*psi));
```

4.3 (a) $f_0 = GCD(60, 160) = 20$ Hz

(b) $T_0 = \frac{1}{f_0} = \frac{1}{20}$ sec

(c) $DC = 11\,e^{j\pi} = -11$

(d)
$$x(t) = -11 + 6e^{-j\frac{\pi}{3}}e^{j120\pi t} + 6e^{j\frac{\pi}{3}}e^{-j120\pi t} + 4e^{-j\frac{\pi}{2}}e^{j320\pi t} + 4e^{j\frac{\pi}{2}}e^{-j320\pi t}$$

$$= -11 + 6e^{-j\frac{\pi}{3}}e^{j(2\pi/T_0)3t} + 6e^{j\frac{\pi}{3}}e^{-j(2\pi/T_0)3t} + 4e^{-j\pi/2}e^{j(2\pi/T_0)8t} + 4e^{j\pi/2}e^{-j(2\pi/T_0)8t}$$

4.3 (d)

k	a_k
0	-11
3	$6e^{-j\pi/3}$
-3	$6e^{j\pi/3}$
8	$4e^{-j\pi/2}$
-8	$4e^{j\pi/2}$

4.4 (a) $x(t)$ is identical to that in Prob. 4.1, i.e.,

$$x(t) = \begin{cases} e^{t} & -1 \le t < 1 \\ 0 & 1 < t < 4 \end{cases}$$

(b) see plot in Prob. 4.1 (a).

(c)
$$a_0 = \frac{1}{5}\left(e - e^{-1}\right) \qquad (\text{see Prob. 4.1 (b)})$$

4.5 (a) $$a_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j(2\pi/T_0)kt}\, dt$$

$$= \frac{1}{5}\int_{-1}^{1} e^{t}\, e^{-j(2\pi/5)kt}\, dt$$

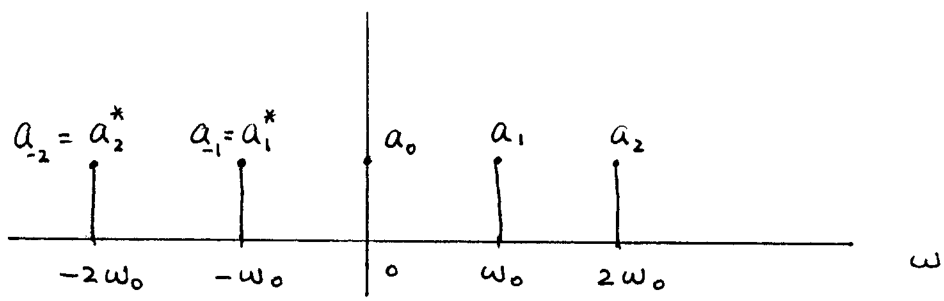
(b) $$a_0 = \frac{1}{5}\int_{-1}^{1} e^{t}\, dt = \frac{1}{5}\left(e - e^{-1}\right)$$

$$a_k = \frac{1}{5}\int_{-1}^{1} e^{\left(1-j\frac{2\pi}{5}k\right)t}\, dt$$

$$= \frac{1}{5}\,\frac{1}{1-j\frac{2\pi}{5}k}\, e^{\left(1-j\frac{2\pi}{5}k\right)t}\Bigg|_{t=-1}^{t=1}$$

$$= \frac{1}{5-j2\pi k}\left(e^{1-j\frac{2\pi}{5}k} - e^{-1+j\frac{2\pi}{5}k}\right)$$

(c)



$$a_0 = \frac{1}{5}\left(e - e^{-1}\right)$$

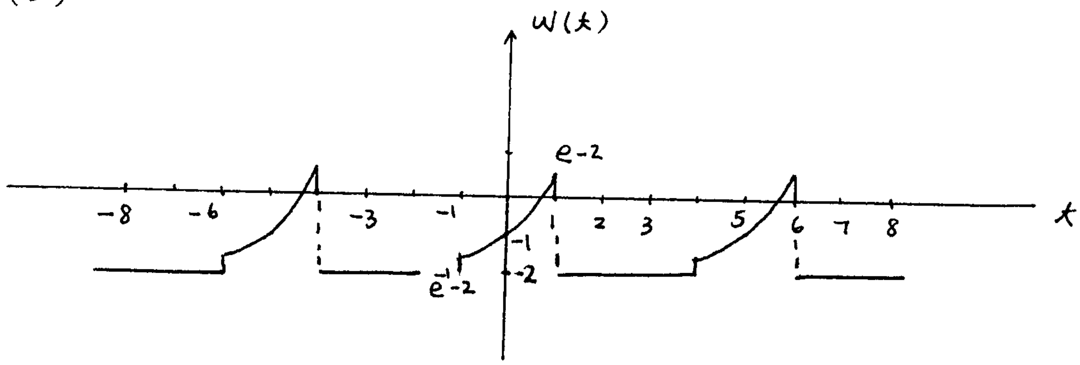
$$a_1 = \frac{1}{5-j2\pi}\left(e^{1-j\frac{2\pi}{5}} - e^{-1+j\frac{2\pi}{5}}\right) = 0.3766\, e^{-j0.4296}$$

$$a_{-1} = a_1^* = 0.3766\, e^{j0.4296}$$

$$a_2 = \frac{1}{5-j4\pi}\left(e^{1-j\frac{4\pi}{5}} - e^{-1+j\frac{4\pi}{5}}\right) = 0.1943\,e^{-j1.1876}$$

$$a_{-2} = a_2^* = 0.1943\,e^{j1.1876}$$

4.6 (a)



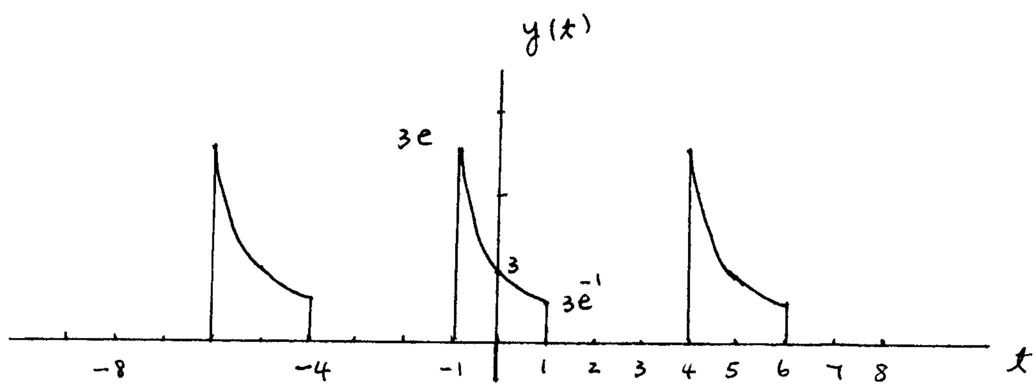
(b) $$w(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t} = x(t) - 2$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} - 2$$

Thus, $b_0 = a_0 - 2$

$b_k = a_k, \quad k \neq 0$

(c)



(d) $$C_k = 3\, a_{-k}$$

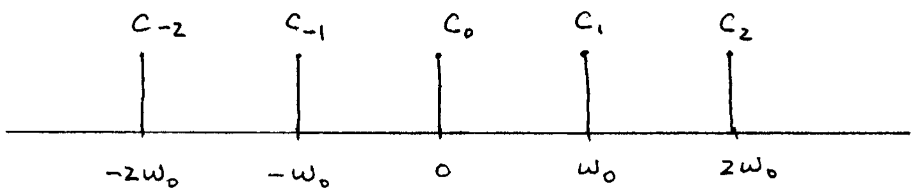
$$C_0 = \frac{3}{5}(e - e^{-1})$$

$$C_1 = 1.1298\, e^{j0.4296}$$

$$C_{-1} = 1.1298\, e^{-j0.4296}$$

$$C_2 = 0.5829\, e^{j1.1876}$$

$$C_{-2} = 0.5829\, e^{-j1.1876}$$



4.7 (a) Because the ratio between the frequencies of same note in successive octaves is 2 and there are 12 notes in each octave. Thus $(2^{1/12})^{12} = 2$ results in the correct relation.

(b)

C	$C^{\#}$	D	E^{b}	E	F	$F^{\#}$	G	$G^{\#}$	A	B^{b}	B	C
40	41	42	43	44	45	46	47	48	49	50	51	52
262	277	294	311	330	349	370	392	415	440	466	494	523

(c) $f = 440 \times 2^{\frac{n-49}{12}}$