ECE-2025 Spring 2004

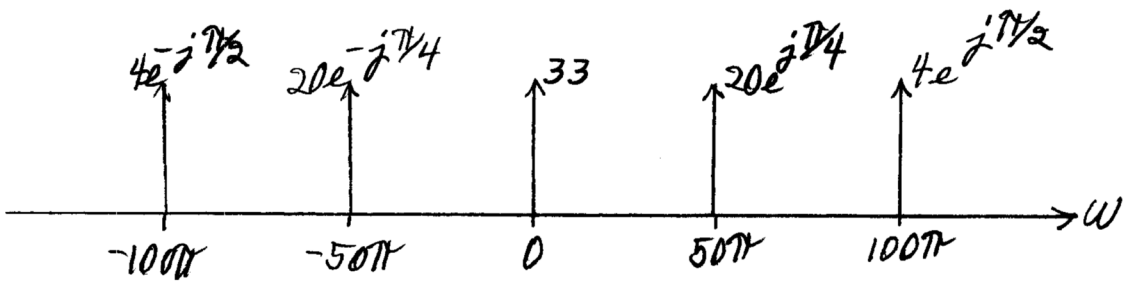
Problem 3.1:

(a)

$$
\begin{aligned}
x(t) &= 5 + 2e^{j3\pi/4}e^{j50\pi t} + 2e^{-j3\pi/4}e^{j(-50\pi)t} \\
&= 5 + 2e^{j(50\pi t + 3\pi/4)} + 2e^{-j(50\pi t + 3\pi/4)} \\
&= 5 + 4\cos(50\pi t + 3\pi/4)
\end{aligned}
$$

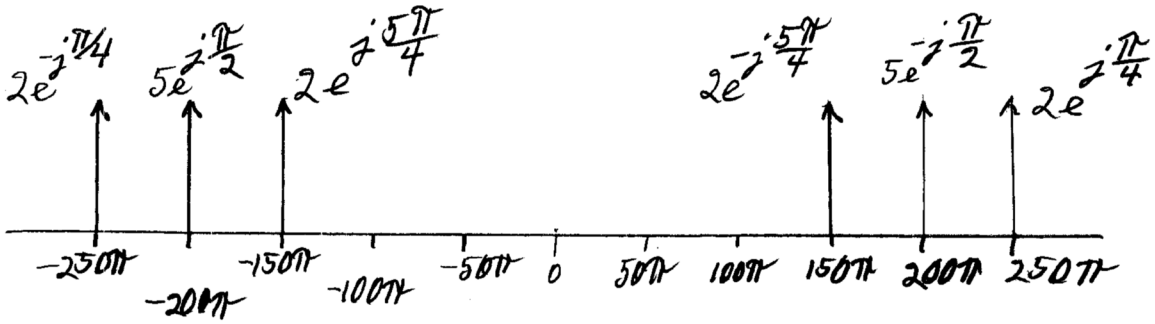
(b)

$$
\begin{aligned}
y(t) &= x^2(t - 0.01) \\
&= (5 + 4\cos(50\pi(t - 0.01) + 3\pi/4))^2 \\
&= (5 + 4\cos(50\pi t - \pi/2 + 3\pi/4))^2 \\
&= (5 + 4\cos(50\pi t + \pi/4))^2 \\
&= \left(5 + 2e^{j(50\pi t + \pi/4)} + 2e^{-j(50\pi t + \pi/4)}\right)^2 \\
&= 25 + 4 + 4 + 20e^{j(50\pi t + \pi/4)} + 20e^{-j(50\pi t + \pi/4)} + 4e^{j(100\pi t + 2\pi/4)} + 4e^{-j(100\pi t + 2\pi/4)} \\
&= 33 + 20e^{j(50\pi t + \pi/4)} + 20e^{-j(50\pi t + \pi/4)} + 4e^{j(100\pi t + \pi/2)} + 4e^{-j(100\pi t + \pi/2)} \\
&= 33 + 20e^{j\pi/4}e^{j50\pi t} + 20e^{-j\pi/4}e^{-j50\pi t} + 4e^{j\pi/2}e^{j100\pi t} + 4e^{-j\pi/2}e^{-j100\pi t}
\end{aligned}
$$



(c)

$$
\begin{aligned}
y(t) &= 2x(t)\sin(200\pi t) \\
&= \left(5 + 2e^{j(50\pi t + 3\pi/4)} + 2e^{-j(50\pi t + 3\pi/4)}\right)\left(\frac{e^{j200\pi t} - e^{-j200\pi t}}{2j}\right) \\
&= \left(5 + 2e^{j(50\pi t + 3\pi/4)} + 2e^{-j(50\pi t + 3\pi/4)}\right)\left(\tfrac{1}{2}e^{-j\pi/2}e^{j200\pi t} + \tfrac{1}{2}e^{j\pi/2}e^{-j200\pi t}\right) \\
&= 2.5e^{-j\pi/2}e^{j200\pi t} + 2.5e^{j\pi/2}e^{-j200\pi t} + e^{j3\pi/4 - j\pi/2}e^{j250\pi t} + e^{j3\pi/4 + j\pi/2}e^{-j150\pi t} \\
&\qquad + e^{-j3\pi/4 - j\pi/2}e^{j150\pi t} + e^{-j3\pi/4 + j\pi/2}e^{-j250\pi t} \\
&= 2.5e^{-j\pi/2}e^{j200\pi t} + 2.5e^{j\pi/2}e^{-j200\pi t} + e^{j\pi/4}e^{j250\pi t} + e^{j5\pi/4}e^{-j150\pi t} + e^{-j5\pi/4}e^{j150\pi t} + e^{-j\pi/4}e^{-j250\pi t}
\end{aligned}
$$



Problem 3.2:

(a)

$$
\begin{aligned}
X_{-5} &= X_5^* = (2e^{j\pi/6})^* = 2e^{-j\pi/6} \\
X_2 &= X_5{-2}^* = (-3 - j\sqrt{3})^* = -3 + j\sqrt{3} = 2\sqrt{3}e^{j5\pi/6} \\
\omega_2 &= -\omega_{-2} = -(-\pi) = \pi \text{ rad/s} \\
\omega_5 &= 2(2.5\omega_2) = 2(\pi) = 2\pi \text{ rad/s}
\end{aligned}
$$

(b)

$$
\begin{aligned}
x(t) &= 4 + 2\sqrt{3}e^{j5\pi/6}e^{j\pi t} + 2\sqrt{3}e^{-j5\pi/6}e^{-j\pi t} + 2e^{j\pi/6}e^{j2.5\pi t} + 2e^{-j\pi/6}e^{-j2.5\pi t} \\
&= 4 + 4\sqrt{3}\cos(\pi t + 5\pi/6) + 4\cos(2.5\pi t + \pi/6)
\end{aligned}
$$

(c) The *fundamental frequency* is $\omega_0 = 0.5\pi$ rad/s, or $f_0 = 0.25$ Hz. This is the greatest common divisor, i.e., $\gcd(\pi, 2.5\pi)$.
Therefore, the *fundamental period* of $x(t)$ is $T_0 = 1/0.25 = 4$ seconds.

Problem 3.3:

3.3(a) $e(t)$ needs to be a sinusoid at $\omega = 2\pi(4)$.

$$x(t) = \mathcal{R}e\left[e^{j(2\pi(436)t+\frac{\pi}{4})} + e^{j(2\pi(440)t+\frac{\pi}{4})} + e^{j(2\pi(444)t+\frac{\pi}{4})}\right]$$

$$= \mathcal{R}e\left[e^{j(2\pi(440)t+\frac{\pi}{4})} e^{j(2\pi(-4)t)} + e^{j(2\pi(440)t+\pi/4)} + e^{j(2\pi(440)t+\frac{\pi}{4})} e^{j2\pi(4)t}\right]$$

$$x(t) = \mathcal{R}e\left[e^{j(2\pi(440)t+\frac{\pi}{4})}\left(1+2\cos 8\pi t\right)\right]$$

$$x(t) = \left(1+2\cos 8\pi t\right)\cos\left(2\pi(440)t+\frac{\pi}{4}\right)$$

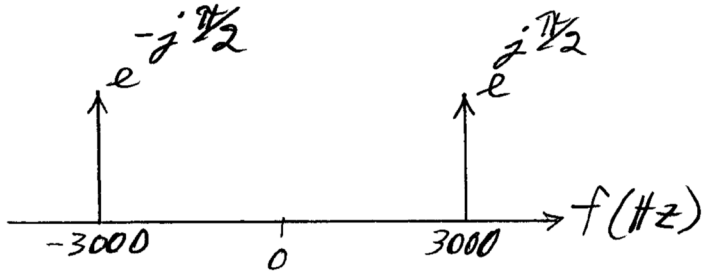
$$\boxed{\therefore\ e(t) = 1+2\cos 8\pi t}$$

3.3(b) $T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{8\pi} = \frac{1}{4}$

Problem 3.4:

3.4 $x(t) = [v(t) + A]\cos 2\pi(750\cdot 10^3)t$

(a) $v(t) = 2\cos\left(2\pi(3000)t + \frac{\pi}{2}\right)$



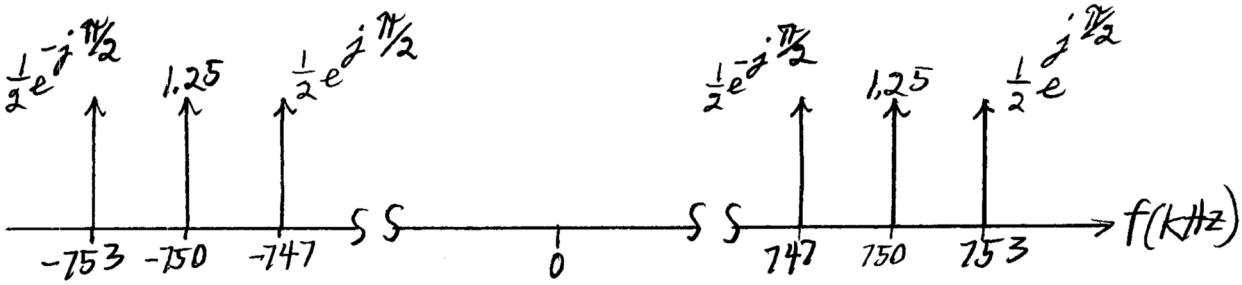
(b) $x(t) = \left[2.5 + e^{j\left[2\pi(3000)t + \frac{\pi}{2}\right]} + e^{-j\left[2\pi(3000)t + \frac{\pi}{2}\right]}\right] \times$

$\frac{1}{2}\left[e^{j2\pi(750\cdot 10^3)t} + e^{-j2\pi(750\cdot 10^3)t}\right]$

$x(t) = 1.25e^{j2\pi(750\cdot 10^3)t} + 1.25e^{-j2\pi(750\cdot 10^3)t}$

$+ \frac{1}{2}e^{j\left[2\pi(753\cdot 10^3)t + \pi/2\right]} + \frac{1}{2}e^{-j\left[2\pi(753\cdot 10^3)t + \pi/2\right]}$

$+ \frac{1}{2}e^{j\left[2\pi(747\cdot 10^3)t - \pi/2\right]} + \frac{1}{2}e^{-j\left[2\pi(747\cdot 10^3)t - \frac{\pi}{2}\right]}$



Problem 3.5:

(a) #5. Period of (a) is 1s; fundamental frequency of #5 is $f_0 = 1$ Hz.

(b) #2. Period of (b) is 1.5s; frequency of #2 is $\omega = 3\pi$ rad/s. Phase of $\phi = -\pi$ means that $x(t)$ is at a negative peak at $t = 0$.

(c) #4. Period of (c) is 2s; fundamental frequency of #4 is $f_0 = 0.5$ Hz.

(d) #1. Period of (d) is 1s, DC value is +2, and $\phi > 0$; frequency of #1 is $f = 1$ Hz with positive phase.

(e) #3. Period of (d) is 1s, DC value is +2, and $\phi < 0$; frequency of #3 is $f = 1$ Hz with negative phase.

1. $x(t) = 2 + 3\cos(2\pi t + \pi/2)$
2. $x(t) = 3\cos(2\pi(1.5)t + \pi)$
3. $x(t) = 2 + 3\cos(2\pi t - 0.4\pi)$
4. $x(t) = 3\cos(\pi t + 0.8\pi) + 3\cos(2\pi(1.5)t + \pi)$
5. $x(t) = 3\cos(2\pi t - 0.4\pi) + 3\cos(4\pi t + \pi)$