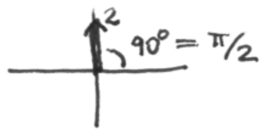
ECE-2025
Spring-2004
Solutions HW #1

PROBLEM 1.1*:

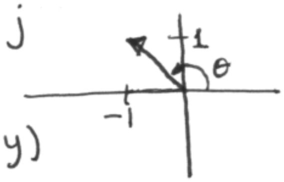
(a) $z = j2$



$z = 2e^{j\pi/2} = 2\angle 90°$

(b) $z = -1 + j$

$(-1, 1) = (x, y)$

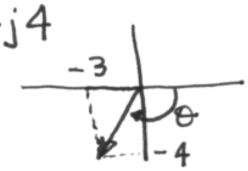


$\theta = 135° = 3\pi/4$ radians

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$z = \sqrt{2}\, e^{j3\pi/4} = \sqrt{2}\, e^{j2.356}$

(c) $z = -3 - j4$



$r = \sqrt{3^2 + 4^2} = 5$ $\quad 5\angle -126.87°$

$\theta = \mathrm{Tan}^{-1}\left(\frac{-4}{-3}\right) = -126.87°$

convert to radians: $-126.87\left(\frac{\pi}{180}\right) = -2.21 = -0.705\pi$

$z = 5e^{-j0.705\pi} = 5e^{-j2.21}$

(d) $z = (0, -1)$

$\theta = -90° = -\pi/2$ rads.

$z = 1e^{-j\pi/2}$

PROBLEM 1.2*:

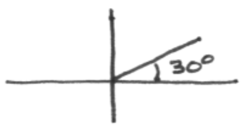
(a) $z=\sqrt{2}\,e^{j3\pi/4}$

$3\pi/4 = 135^\circ$

$z=\sqrt{2}\left(\cos\frac{3\pi}{4}+j\sin\frac{3\pi}{4}\right)=\sqrt{2}\left(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)=-1+j1$

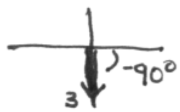
(b) $z=1.6\angle\pi/6 = 1.6\,e^{j\pi/6} = 1.6\left(\cos\frac{\pi}{6}+j\sin\pi/6\right)$

$=1.6\angle 30^\circ$



$=1.6\left(\frac{\sqrt{3}}{2}+j\frac{1}{2}\right)=1.386+j0.8$

(c) $z=3e^{-j\pi/2}=3\angle -90^\circ$



$z=-3j$

(d) $z=7\angle 7\pi = 7\angle\pi = 7e^{j\pi} = -7+j0$ (subtract multiple of 2π)

$=7\angle 1260^\circ$

PROBLEM 1.3*:

(a) $z = -3 + j4 = 5e^{j0.705\pi}$ (ANGLE $= 126.87^\circ$ or 2.214 rads)

$\frac{1}{z} = \frac{1}{5} e^{-j0.705\pi}$

(b) $z = -2 + j2 = 2\sqrt{2}\, e^{j3\pi/4}$

SUBTRACT 4π

$z^5 = (2\sqrt{2})^5 e^{j15\pi/4} = 128\sqrt{2}\, e^{-j\pi/4}$

(c) $z = -5 + j13$

$|z|^2 = z z^* = (-5 + j13)(-5 - j13)$

$= 25 + 169 = 194$

(d) $\Re e\{(-2+j5)e^{-j\pi/2}\}$

EQUALS $-j$

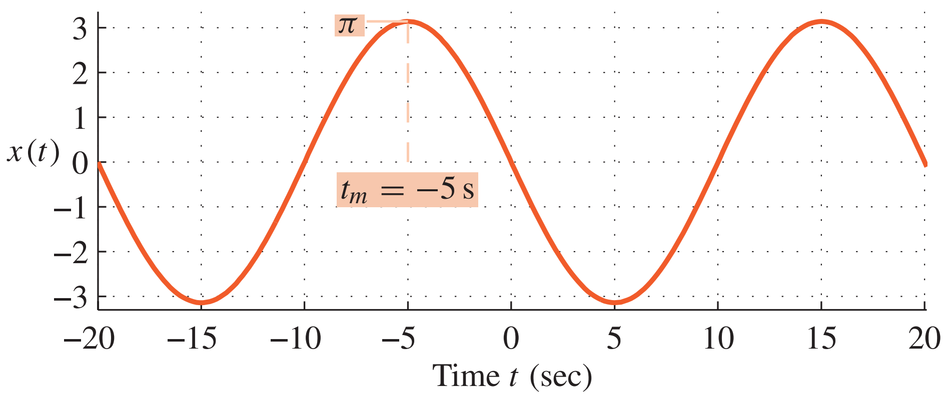
$= \Re e\{(-2+j5)(-j)\}$

$= \Re e\{2j + 5\} = 5$

PROBLEM 1.4*:
Plot two periods of the following sinusoids with $t = 0$ in the middle (i.e., for $-T \le t \le T$):

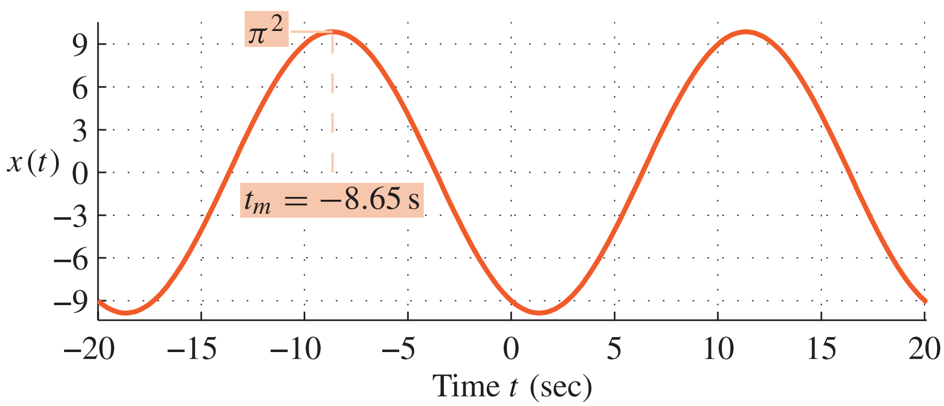
(a) $x(t) = \pi\cos((\pi/10)t + 0.5\pi)$

Solution:
The period is $T = 2\pi/\omega_0 = 2\pi/(\pi/10) = 20$ s.
The time of a maximum is $t_m = -\phi/\omega_0 = -0.5\pi/(\pi/10) = 5$ s.



(b) $x(t) = \pi^2\cos((\pi/10)t + e)$

Solution:
The period is $T = 2\pi/\omega_0 = 2\pi/(\pi/10) = 20$ s.
The time of a maximum is $t_m = -\phi/\omega_0 = -e/(\pi/10) = -2.718/(\pi/10) = 8.65$ s.

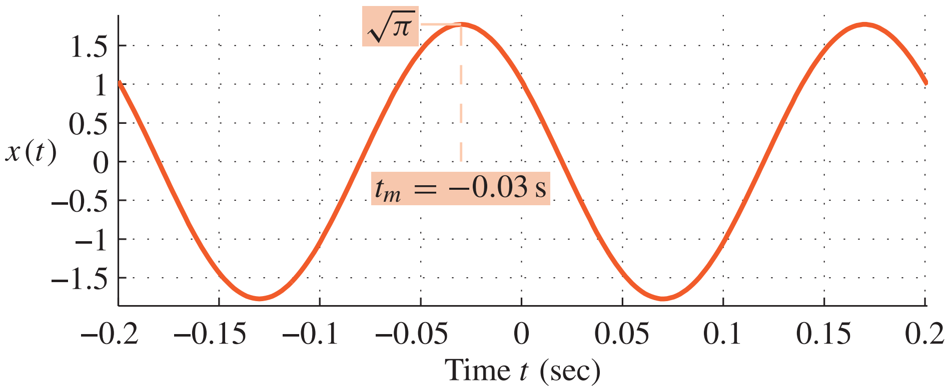


(c) $x(t) = \sqrt{\pi}\cos(10\pi t - 7.7\pi)$

Solution:

The period is $T = 2\pi/\omega_0 = 2\pi/(10\pi) = 0.2$ s.

The time of a maximum is $t_m = -\phi/\omega_0 = -(-7.7\pi)/(10\pi) = 0.77$ s. If we want to get the peak closest to $t = 0$, then we must reduce ϕ modulo–2π so that it is within the interval $-\pi < \phi \leq \pi$. We can do this by adding a multiple of 2π to ϕ; in this case, $4(2\pi) - 7.7\pi = 0.3\pi$. Then $t_m = -\phi/\omega_0 = -(0.3\pi)/(10\pi) = -0.03$ s.



PROBLEM 1.4*:

The waveform in the figure generated from the MATLAB GUI `sindrill` can be expressed as

$$x(t) = A\cos[\omega_0(t - t_d)] = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

From the waveform, it is easy to see that $A = 10$ and the period is $T = 0.01$ s. Furthermore, the time of the positive maximum nearest to $t = 0$ is $t_m \approx -0.004$ s.

From these measurements, we can calculate the frequency

$$\omega_0 = 2\pi/T = 2\pi/(0.01) = 200\pi \text{ rad/s, or } f_0 = 100 \text{ Hz}$$

and the time delay $t_d = t_m = -0.004$ s; and the phase

$$\phi = -\omega T = -(200\pi)(-0.004) = 0.8\pi \text{ radians}$$

This value of ϕ satisfies $-\pi < \phi \leq \pi$.

Note: The true value of the phase is $\phi = 0.75\pi$, but it would be hard to get that exact value from the plot.