Problem Set #3

Solution #1

a) $$x(t) = 4\cos(\pi t - \pi/8) - \cos(\sqrt{3}\pi t + \pi/4) + \sin(3\pi t)$$

Recall that

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \quad \& \quad \sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

Thus,

$$x(t) = 4 \cdot \frac{1}{2} \cdot \left[e^{j(\pi t - \pi/8)} + e^{-j(\pi t - \pi/8)} \right]$$

$$- \frac{1}{2}\left[e^{j(\sqrt{3}\pi t + \pi/4)} + e^{-j(\sqrt{3}\pi t + \pi/4)} \right]$$

$$+ \frac{1}{2j}\left[e^{j3\pi t} - e^{-j3\pi t} \right]$$

$$= 2e^{-j\pi/8} \cdot e^{j\pi t} + 2 \cdot e^{j\pi/8} \cdot e^{-j\pi t}$$

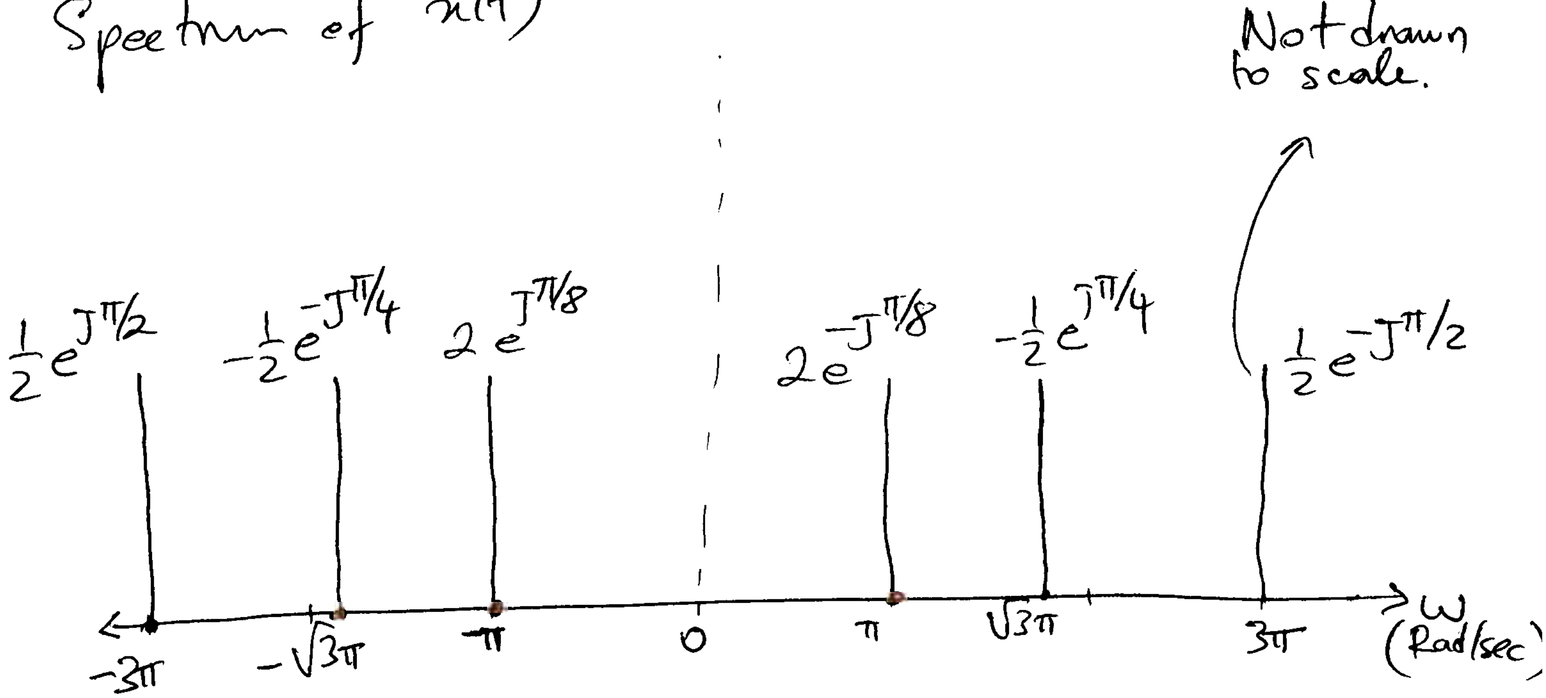
$$- \frac{1}{2}e^{j\pi/4} \cdot e^{j\sqrt{3}\pi t} - \frac{1}{2}e^{-j\pi/4} \cdot e^{-j\sqrt{3}\pi t}$$

$$1/j \qquad\qquad -\frac{1}{j}$$

$$+ \frac{1}{2}e^{-j\pi/2} \cdot e^{j3\pi t} + \frac{1}{2}e^{j\pi/2} \cdot e^{-j3\pi t}$$

Easier way $\sin(13\pi t) = \cos(13\pi t - \pi/2)$ & proceed similarly.

Spectrum of $x(t)$



b) Freq of $\cos(\pi t - \pi/8) = \frac{1}{2} \longleftrightarrow T_1 = 2$

Freq of $\cos(\sqrt{3}\pi t + \pi/4) = \frac{\sqrt{3}}{2} \longleftrightarrow T_2 = 2/\sqrt{3}$

Freq of $\sin(3\pi t) = 3/2. \longleftrightarrow T_3 = 2/3$

$\frac{T_1}{T_2}$ is not a rational #, $x(t)$ is not periodic.

Solution #2

a) $x(t) = 16 + 8e^{j\pi/7} \cdot e^{j(160\pi)t} + 8e^{-j\pi/7} \cdot e^{j(-160\pi)t}$
$+ 14e^{-j\pi/3} \cdot e^{j(240\pi)t} + 14e^{j\pi/3} \cdot e^{j(-240\pi)t}.$

$= 16 + 16\cos(160\pi t + \pi/7) + 28\cos(240\pi t - \pi/3)$

b) Period of $\cos(160\pi t + \pi/7) = 1/80 = T_2 = \frac{3}{240}$

Period of $\cos(240\pi t - \pi/3) = \cancel{1/120} = \frac{1}{120} = T_3 = \frac{2}{240}$

Smallest $n, m \in \mathbb{Z}$ (Integer) such that $n T_2 = m T_3$ are $n=2$, $m=3$. Periodic.

Thus, fundamental period is $2 \cdot \frac{3}{240} = 3 \cdot \frac{2}{240} = \frac{1}{40}$

Solution #3

a) $x(t) = 2\cos(416\pi t + \pi/3)$

$y(t) = x(t)^3 = 8\cos^3(416\pi t + \pi/3)$

$= 8\left[\frac{e^{j(416\pi t + \pi/3)} + e^{-j(416\pi t + \pi/3)}}{2}\right]^3$

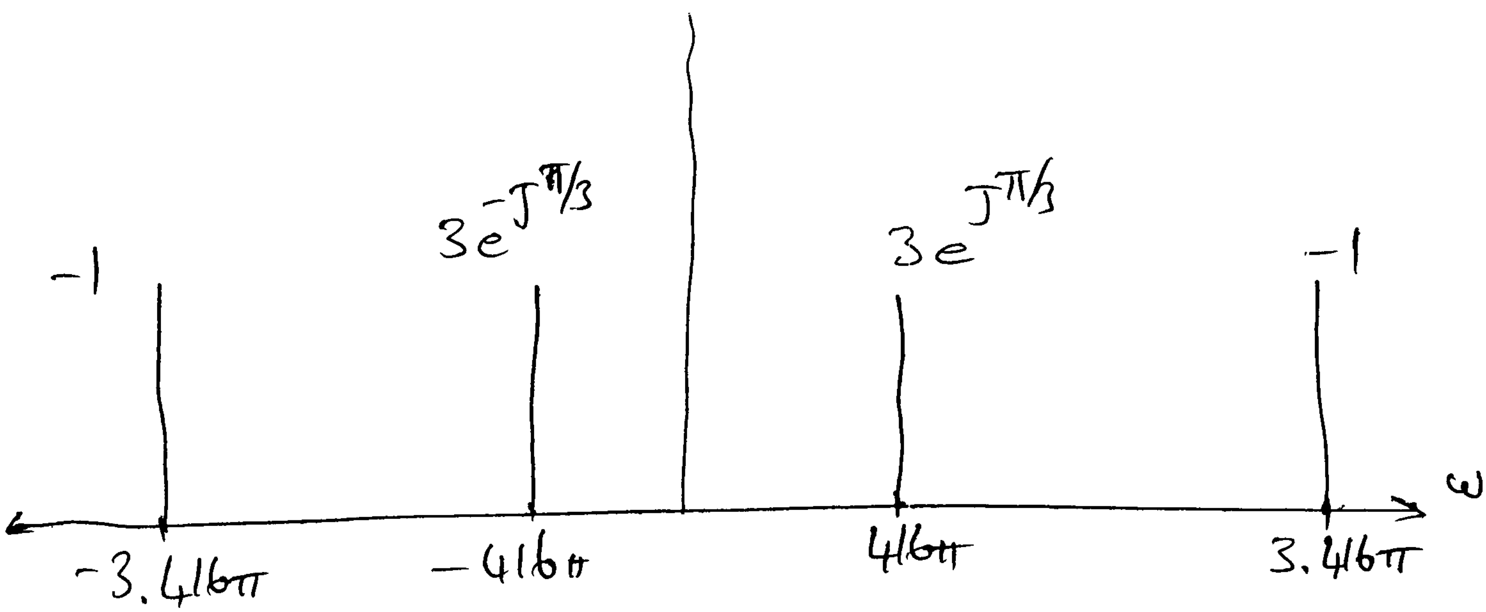
Recall that $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$.

$$y(t) = e^{j\pi} \cdot e^{j\,3.416\pi t} + 3e^{j\frac{2\pi}{3}} \cdot e^{j\,2.416\pi t} \cdot e^{-j\pi/3} \cdot e^{j\,416\pi t}$$

$$+\,3e^{j\pi/3} \cdot e^{j\,416\pi t} \cdot e^{-j\,2\pi/3} \cdot e^{-j\,2.416\pi t} + e^{-j\pi} \cdot e^{-j\,3.416\pi t}$$

$$\Rightarrow y(t) = e^{j\pi} e^{j\,3.416\pi t} + e^{-j\pi} \cdot e^{-j\,3.416\pi t} +$$

$$3e^{j\pi/3} \cdot e^{j\,416\pi t} + 3e^{-j\pi/3} \cdot e^{-j\,416\pi t}$$



b) $$y(t) = 6\cos(416\pi t + \pi/3) - \cos(3.416\pi t)$$

c) $$T_1 = \frac{2}{416} \qquad T_2 = \frac{2}{3.416}.$$

Find $n, m \ni nT_1 = mT_2. \Rightarrow n=1,\ m=3$

Periodic. Fundamental Period is $nT_1 = 2/416 = \underline{\underline{1/208}}$

Solution #4

a) $x(t) = \cos(2\pi \cdot 262t) + \cos(2\pi \cdot 268t)$

$$= \mathrm{Re}\{e^{j \cdot 2\pi \cdot 262t}\} + \mathrm{Re}\{e^{j \cdot 2\pi \cdot 268t}\}$$

$$= \mathrm{Re}\{e^{j \cdot 2\pi \cdot 262t} + e^{j \cdot 2\pi \cdot 268t}\}$$

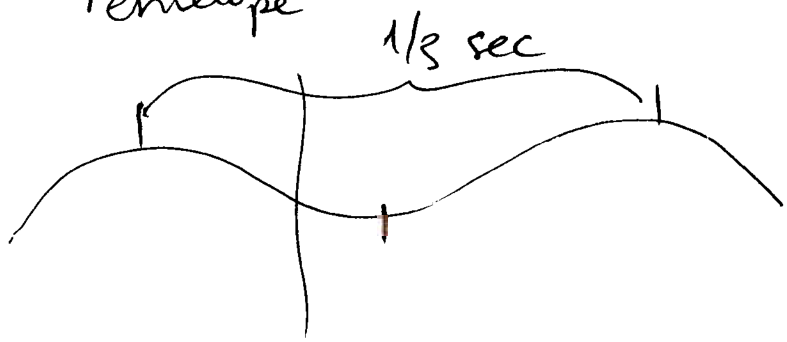
$$= \mathrm{Re}\{e^{j \cdot 2\pi \cdot 265t}(e^{-j \cdot 2\pi \cdot 3t} + e^{+j \cdot 2\pi \cdot 3t})\}$$

$$= \mathrm{Re}\{2\cos(2\pi \cdot 3t) \cdot e^{j \cdot 2\pi \cdot 265t}\}$$

$$= 2\cos(2\pi \cdot 3t) \cdot \cos(2\pi \cdot 265t)$$

$\underline{f_c = 265 \text{ Hz}}$ (Short-cut $= \frac{262+268}{2}$ Hz)

b). $f_{envelope} = 3$ Hz $\Rightarrow T = 1/3$ sec.



Between the nulls, we have $\frac{1}{2} \cdot \frac{1}{3} = \underline{1/6 \text{ sec.}}$

c) $T_1 = 1/262$ & $T_2 = 1/268$

$$\frac{T_1}{T_2} = \frac{268}{262} = \frac{134}{131}$$

$x(t)$ is periodic. Fundamental period is

$$\underbrace{134 \cdot T_2 = 131 \cdot T_1}_{\text{Harmonics}} = \underbrace{\tfrac{1}{2} \text{ sec}}_{\text{period}}.$$

<u>Solution 5</u>

$$x(t) = [12 + 7\,\mathrm{Sin}(\pi t - \pi/3)] \cdot \mathrm{Cos}(13\pi t)$$

$$= 12\,\mathrm{Cos}(13\pi t) + 7\,\mathrm{Sin}(\pi t - \pi/3)\,\mathrm{Cos}(13\pi t)$$

$$= 12\,\mathrm{Cos}(13\pi t) + \mathrm{Re}\left\{7\,\mathrm{Cos}(\pi t - 5\pi/6)\, e^{j13\pi t}\right\}$$

$$= 12\,\mathrm{Cos}(13\pi t) + \mathrm{Re}\left\{\left(\frac{7}{2} \cdot e^{j(\pi t - 5\pi/6)} + \frac{7}{2} e^{-j(\pi t)-5\pi/6)}\right) \cdot e^{j13\pi t}\right\}$$

$$= 12\,\mathrm{Cos}(13\pi t) + \mathrm{Re}\left\{\frac{7}{2} \cdot e^{-j5\pi/6} \cdot e^{j14\pi t}\right\}$$

$$+ \mathrm{Re}\left\{\frac{7}{2} \cdot e^{+j5\pi/6} \cdot e^{+j12\pi t}\right\}$$

$$= 12\,\mathrm{Cos}(13\pi t) + \frac{7}{2} \cdot \mathrm{Cos}(14\pi t - 5\pi/6) + \frac{7}{2}\,\mathrm{Cos}(12\pi t + 5\pi/6)$$

Thus,

$A_1 = 7/2$ $\qquad \omega_1 = 5\pi/6$

$A_2 = 12$ $\qquad \omega_2 = 0$

$A_3 = 7/2$ $\qquad \omega_3 = -5\pi/6.$

Solution #6.

$a \longleftrightarrow 3$

$b \longleftrightarrow 5$

$c \longleftrightarrow 1$

$d \longleftrightarrow 2$

$e \longleftrightarrow 4$