ECE2025 Fall 2003
Solution of Problem Set #2

Problem 2.1

(a)

$$x_a(t) = 2\cos(100\pi t + 3\pi/4) + \sqrt{2}\cos(100\pi t)$$

$$\Updownarrow$$

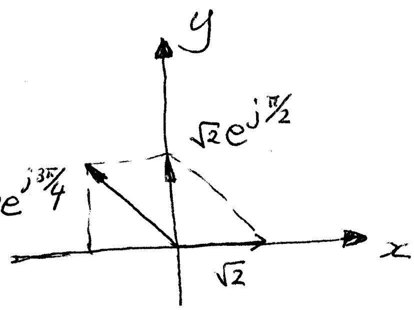
$$X = 2e^{j3\pi/4} + \sqrt{2}$$

$$= j\sqrt{2}$$

$$= \sqrt{2}e^{j\pi/2}$$

$$\Updownarrow$$

$$x_a(t) = \sqrt{2}\cos(100\pi t + \pi/2)$$



(b)

$$x_b(t) = 4\cos(2000\pi t + 7\pi) + 5.5\cos(2000\pi t - 2.5\pi) - 6\cos(2000\pi t - 3\pi/4)$$

$$\Updownarrow$$

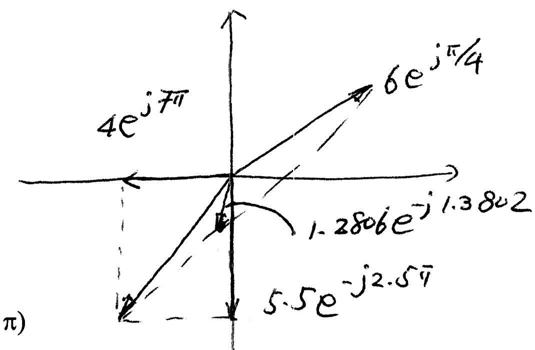
$$X = 4e^{j7\pi} + 5.5e^{-j2.5\pi} - 6e^{-j3\pi/4}$$

$$\approx 0.2426 - j1.2574$$

$$\approx 1.2806e^{-j1.3802}$$

$$\Updownarrow$$

$$x_b(t) = 1.280\cos(100\pi t - 1.3802)$$



(c)

$$x_b(t) = 50\cos(120\pi t - \pi/6) + 50\cos(120\pi t - 5\pi/6) + 50\sin(120\pi t + \pi)$$

$$= 50\cos(120\pi t - \pi/6) + 50\cos(120\pi t - 5\pi/6) + 50\cos(120\pi t + \pi/2)$$

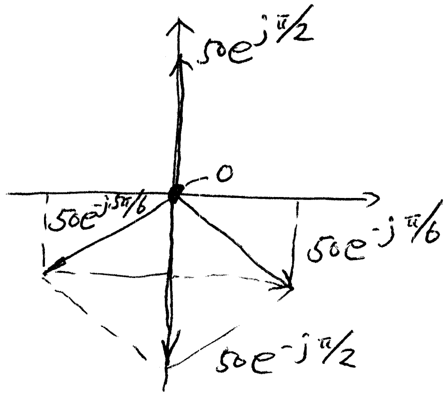
$$\Updownarrow$$

$$X = 50e^{-j\pi/6} + 50e^{-j5\pi/6} + 50e^{j\pi/2}$$

$$\approx 0$$

$$\Updownarrow$$

$$x_c(t) = 0$$



Problem 2.2

(a)

$$\frac{d}{dt}z(t) = \underbrace{Z \bullet (j40\pi)}_{Q} \bullet e^{j40\pi t}$$

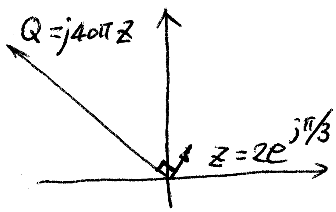
$$= Qe^{j40\pi t}$$

$$Q = Z \bullet (j40\pi)$$

$$= 40\pi e^{j\pi/2}Z$$

$$= 80\pi e^{j5\pi/6}$$

(b) The angle of Q is $\pi/2$ greater than that of Z. It is also shown by the following figure.



(c) Since $z(t) = 2e^{j(40\pi t+\pi/3}$ and

$$\Re e\left\{\frac{d}{dt}z(t)\right\} = \Re e\left\{(j40\pi)\bullet 2e^{j(40\pi t+\pi/3)}\right\}$$
$$= -80\pi\sin(40\pi t+\pi/3)$$
$$\frac{d}{dt}\Re e\{z(t)\} = \frac{d}{dt}2\cos(40\pi t+\pi/3)$$
$$= -40\pi\bullet 2\sin(40\pi t+\pi/3)$$
$$= \Re e\left\{\frac{d}{dt}z(t)\right\}$$

It is true for all complex exponential signals.

(d)

$$\int_{-0.05}^{0.05} z(t)dt = \int_{-0.05}^{0.05} Ze^{j40\pi t}dt$$
$$= \frac{1}{j40\pi}Ze^{j40\pi t}\Big|_{t=-0.05}^{0.05}$$
$$= \frac{1}{j40\pi}Ze^{j40\pi\bullet 0.05} - \frac{1}{j40\pi}Ze^{j40\pi t\bullet(-0.05)}$$
$$= 0$$

(e)

$$z(t-t_d) = Ze^{j40\pi(t-t_d)}$$
$$= \underbrace{Ze^{-j40\pi t_d}}_{P}e^{j40\pi t}$$
$$P = Ze^{-j40\pi t_d} = 2e^{-j40\pi t_d + j\pi/3}$$

$z(t-t_d) = z(t)$ if and only if $e^{-j40\pi t_d} = 1$ or $40\pi t_d = 2\pi k$ for any integer k, that is, $t_d = k/20$ for $k = 0, \pm 1, \pm 2, \cdots$.

Problem 2.3

(a) From the figure, we can see that $A = 20$, $t_d = 0.01$ sec, and $T = 0.02$. Therefore,

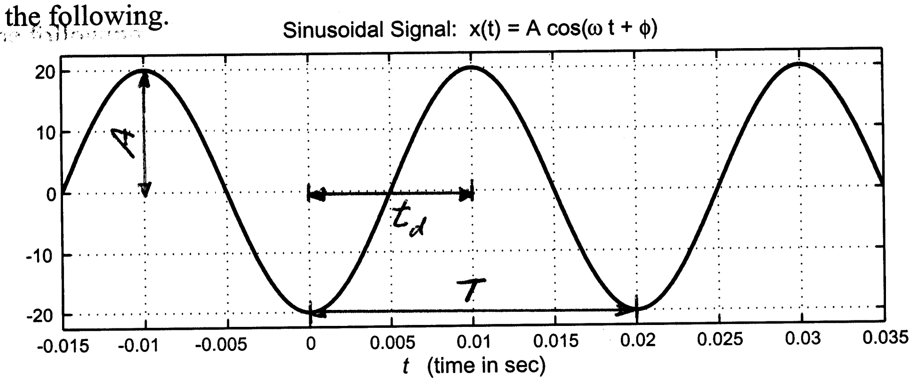
$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{0.02} = 100\pi,$$
$$\phi = -\omega_o t_d = -100\pi\bullet 0.01 = -\pi,$$
$$x(t) = 20\cos(100\pi t - \pi).$$

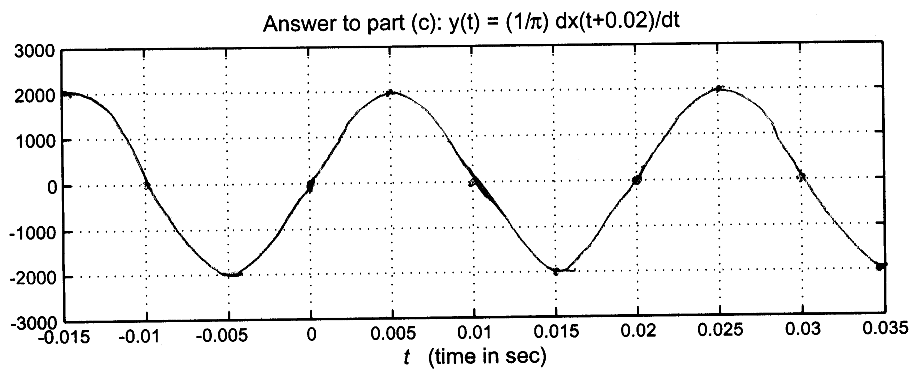
(b)

$$
\begin{aligned}
x(t) &= 20\cos(100\pi t - \pi) \\
&= \Re e\left\{20 e^{j(100\pi t-\pi)}\right\} \\
&= \Re e\left\{\underbrace{20 e^{-j\pi}}_{Z} e^{j100\pi t}\right\},
\end{aligned}
$$

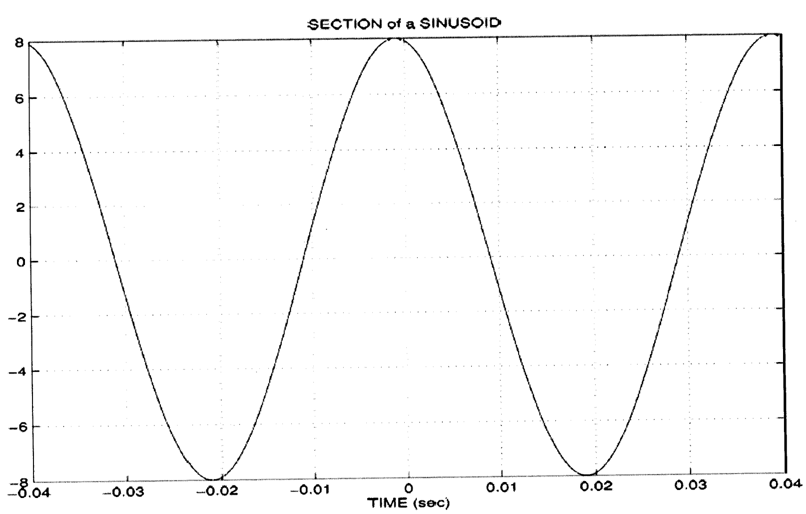
$$Z = 20 e^{-j\pi}.$$

(c) $y(t) = \frac{1}{\pi}\frac{d}{dt}[x(t+0.02)] = \frac{1}{\pi}\frac{d}{dt}[20\cos(100\pi(t+0.02)-\pi)] = 2000\cos(100\pi t - \pi/2)$. The figure is shown as the following.





Problem 2.4 $A = 8$, $\omega_o = 2\pi F_o$, $T = 1/F_o = 0.04$, and $\phi = \omega_0 t_d = 2\pi \bullet 25 \bullet 0.001 = 0.05\pi$. The figure is as the following.



Problem 2.5 Substitute a by $j\theta$ in the Taylor series of $\exp(a)$, we have

$$\begin{aligned}\exp(j\theta) &= 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \cdots \\ &= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} + \cdots \\ &= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots\right) + j\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots\right) \\ &= \cos(\theta) + j\sin(\theta),\end{aligned}$$

which proves Euler's formula.