GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 11-July-03 COURSE: ECE 2025

NAME: Key
LAST, FIRST

STUDENT #: ____________

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-10:00am (D. Taylor)

L02:Tues-12:00am (T. Michaels)

L03:Tues-2:00pm (D. Taylor)

L04:Tues-4:00pm (D. Taylor)

L06:Mon-4:00pm (T. Michaels)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive any partial credit. Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem Q3.1:

Suppose a discrete-time filter has the frequency response

$$H_1(e^{j\hat{\omega}}) = \cos(3\hat{\omega})\, e^{-4j\hat{\omega}}.$$

(a) If the signal

$$x[n] = 3 + 5\cos\left(\frac{\pi}{12}n + \pi\right) \quad \text{for all integer } n.$$

is input to the system, find a simple mathematical expression for the output signal $y[n]$.

$|H_1(e^{j0})| = 1$

$|H_1(e^{j\frac{\pi}{12}})| = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$, $\angle H_1(e^{j\frac{\pi}{12}}) = -\frac{\pi}{3}$

$y[n] = 3 + \frac{5}{\sqrt{2}}\cos(\frac{\pi}{12}n + \pi - \frac{\pi}{3})$

$\boxed{= 3 + \frac{5}{\sqrt{2}}\cos(\frac{\pi}{12}n + \frac{2\pi}{3})}$

or $\boxed{\approx 3 + 3.5355\cos(\frac{\pi}{12}n + \frac{2\pi}{3})}$

(b) We could implement the above system in MATLAB for some arbitrary finite input sequence x using the command y = conv(b,x). Find the appropriate filter coefficient vector b corresponding to the system with the frequency response $H_1(e^{j\hat{\omega}})$ given above.

$H_1(e^{j\hat{\omega}}) = \frac{1}{2}[e^{3j\hat{\omega}} + e^{-3j\hat{\omega}}]e^{-4j\hat{\omega}}$

$= \frac{1}{2}e^{-j\hat{\omega}} + \frac{1}{2}e^{-7j\hat{\omega}}$

$b = [0 \ \frac{1}{2} \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{2}]$

(c) Now suppose we instead have a system with frequency response

$$H_2(e^{j\hat{\omega}}) = \frac{\sin(3\hat{\omega})}{\sin(\hat{\omega}/2)}e^{-5j\hat{\omega}/2}.$$

Now what should b be in order to implement this system with the MATLAB command y = conv(b,x)? (You should be able to find the answer without much work at all.)

This Is a 6-PT ~~Runninger~~, Sum,

So $b = [1 \ 1 \ 1 \ 1 \ 1 \ 1]$

Problem Q3.2:

In each of the following cases, simplify the expression using the properties of the continuous-time unit impulse signal. Provide some explanation or intermediate steps for each answer.

(a) $\frac{d}{dt}\left\{\sin\left(\frac{\pi}{2}t\right)u(t-3)\right\} = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)u(t-3) + \sin\left(\frac{\pi}{2}t\right)\delta(t-3)$

$= \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)u(t-3) + \sin\left(\frac{3\pi}{2}\right)\delta(t-3)$

$= \boxed{\frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)u(t-3) - \delta(t-3)}$

(b) $\int_{-\infty}^{t-5} \delta(\tau+3)e^{4\tau}\,d\tau = \int_{-\infty}^{t-5} \delta(\tau+3)e^{-12}\,d\tau$

$= e^{-12}u(t-5+3)$

$= \boxed{e^{-12}u(t-2)}$

Problem Q3.3:

Suppose $h(t) = \arctan(t)[u(t-1) - u(t-3)]$ and $x(t) = \cos^2(t)[u(t-2) - u(t-7)]$. Fill in the boxes in the following first step of computing the convolution of $h(t)$ with $x(t)$, where we are choosing to let $h(t)$ be the function that we are flipping and shifting:

$$(h * x)(t) = \begin{cases} 0 & \text{for } t \leq \boxed{3} \\ \int_{\boxed{2}}^{\boxed{t-1}} \arctan(t-\tau)\cos^2(\tau)d\tau & \text{for } \boxed{3} < t \leq \boxed{5} \\ \int_{\boxed{t-3}}^{\boxed{t-1}} \arctan(t-\tau)\cos^2(\tau)d\tau & \text{for } \boxed{5} < t \leq \boxed{8} \\ \int_{\boxed{t-3}}^{\boxed{7}} \arctan(t-\tau)\cos^2(\tau)d\tau & \text{for } \boxed{8} < t \leq \boxed{10} \\ 0 & \text{for } t > \boxed{10} \end{cases}$$

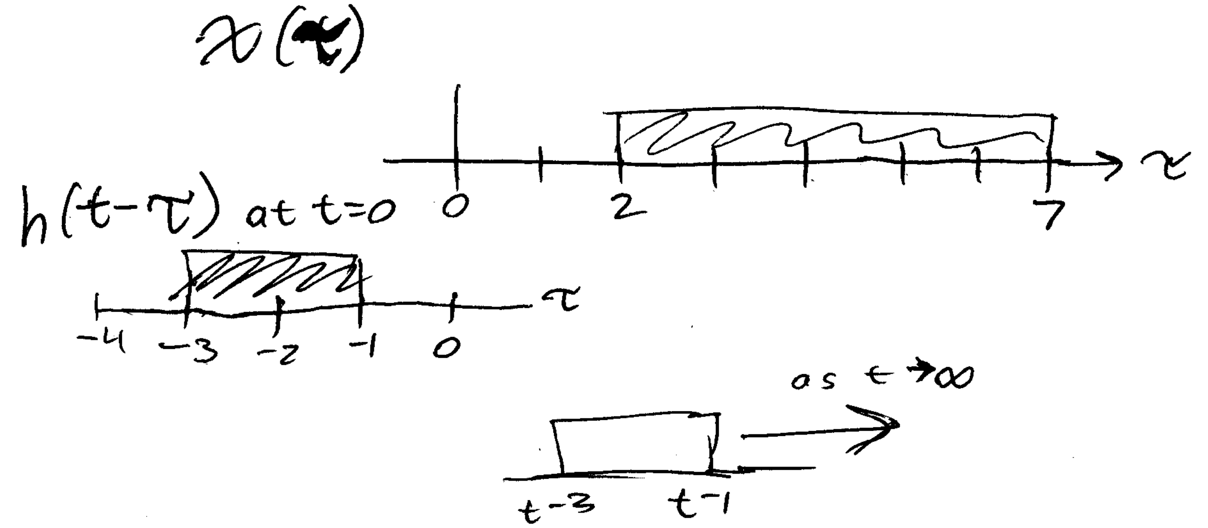
$t-1<2$

$t-3>2$

$t-1<7 \rightarrow$

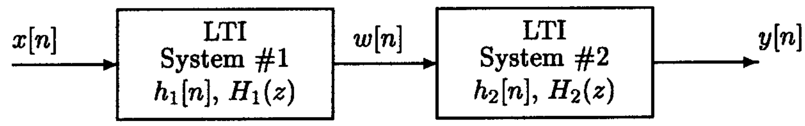
$t-3>7$

Just fill in the boxes. Don't try to work any more of the convolution, i.e. don't try actually working out the integrals! (Notice we picked arctan and $\cos^2$ rather arbitrarily. The point of the problem is to see if you can figure out the limits of the integrals and what the different regions are.)



Problem Q3.4:

A cascade of two discrete-time systems is depicted by the following block diagram:



System #1 is defined by the system function $H_1(z) = 1 - 0.5z^{-1}$ and System #2 is defined by the difference equation $y[n] = 0.9y[n-1] + w[n-1]$.

(a) If the input to the first system is $x[n] = \delta[n] + \delta[n-1]$, determine the output, $w[n]$, of the first system.

$$X(z) = 1 + z^{-1}$$

$$W(z) = X(z)H_1(z) = (1 - \tfrac{1}{2}z^{-1})(1 + z^{-1})$$

$$= 1 + (1 - \tfrac{1}{2})z^{-1} - \tfrac{1}{2}z^{-2} = 1 + \tfrac{1}{2}z^{-1} - \tfrac{1}{2}z^{-2}$$

$$w[n] = \delta[n] + \tfrac{1}{2}\delta[n-1] - \tfrac{1}{2}\delta[n-2]$$

(Could also do this problem by finding $h_1[n]$ and working the convolution table)

(b) Determine the system function, $H(z)$, of the overall system.

$$Y(z) = 0.9z^{-1}Y(z) + z^{-1}W(z)$$

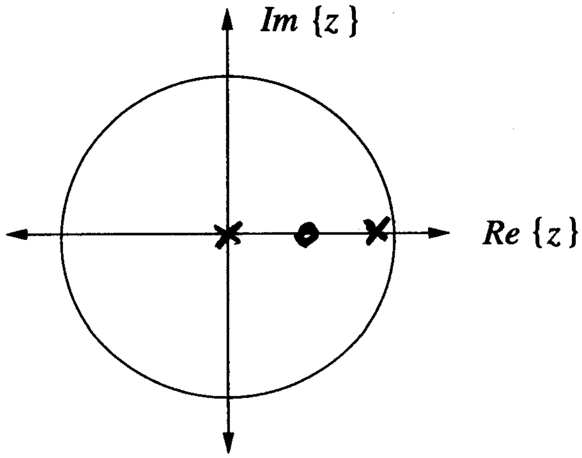
$$Y(z)[1 - 0.9z^{-1}] = z^{-1}W(z)$$

$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{z^{-1}}{1 - 0.9z^{-1}}$$

$$H(z) = H_1(z)H_2(z) = (1 - 0.5z^{-1})\frac{z^{-1}}{1 - 0.9z^{-1}}$$

$$= \frac{z^{-1} - 0.5z^{-2}}{1 - 0.9z^{-1}}$$

(c) Plot the poles and zeros of $H(z)$.



$$H(z) = \frac{(z-0.5)}{(z-0.9)z}$$

(d) Determine the impulse response, $h[n]$, of the overall system.

$$h[n] = (0.9)^{(n-1)} u(n-1) - 0.5 \times (0.9)^{n-2} u(n-2)$$

From

$$a^n u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-az^{-1}}$$

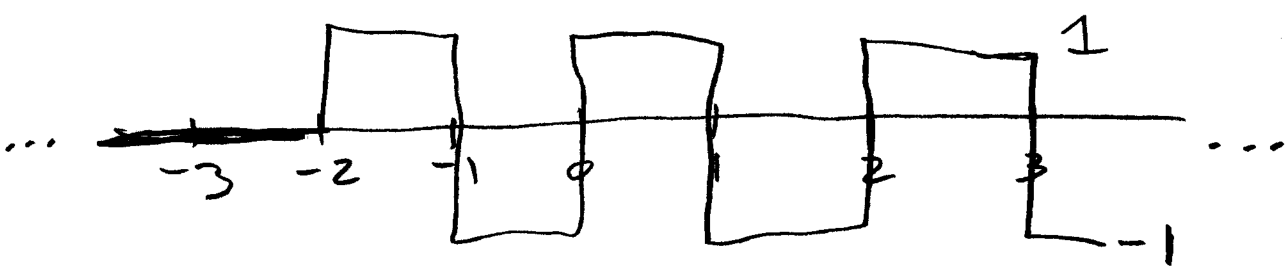
Problem Q3.5:

Suppose a continuous-time LTI system has an impulse response specified by this MATLAB function:

```
function output = h(t);
h1 = sin(pi*t)
h2 = h1 ./ abs(h1);
output = h2 .* (t > -2)
```

Notice that depending on t, $h(t)$ takes on values 0, -1, or 1.

(a) Draw a carefully labeled sketch of $h(t)$.



(b) Notice that for all a,

$$\int_{-\infty}^{a} h(t)dt < \infty$$

A fellow student tells you that this implies that $h(t)$ is the impulse response of a stable system. Is your fellow student correct? If not, what is wrong with his reasoning?

No! The condition is

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

student forgot the absolute value

(c) Suppose the system represented by the impulse response $h(t)$ is cascaded with another system with impulse response $g_1(t) = \delta(t - b)$.

For what values of b is the complete cascaded system causal?

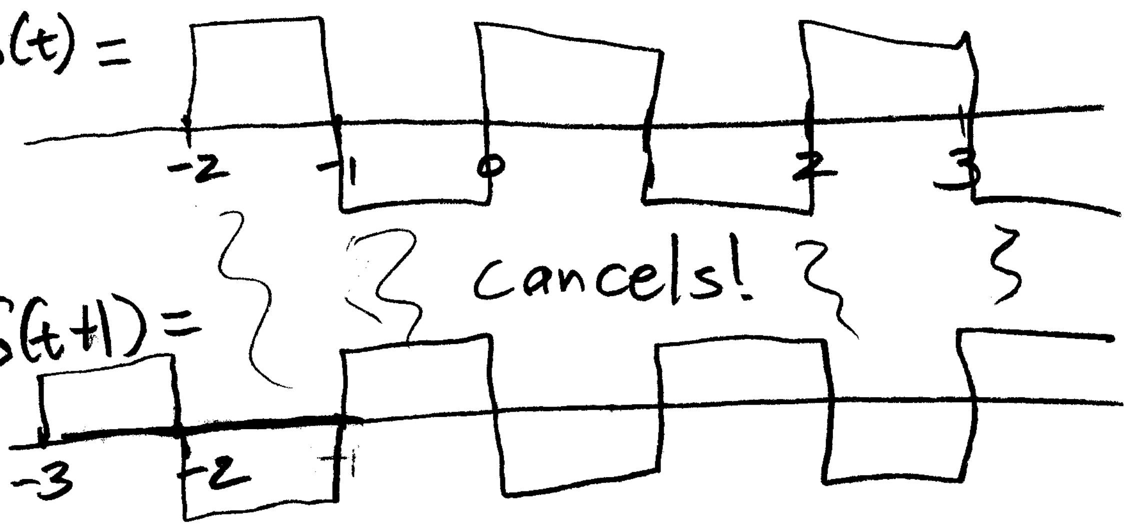
Need $b \geq 2$

since waveform starts at $t = -2$;
must shift 2 or more to the right

(d) Now suppose the system represented by the impulse response $h(t)$ is cascaded with another system with impulse response $g_2(t) = \delta(t) + \delta(t+1)$. Sketch $h_{cas}(t) = (h * g_2)(t)$, which is the impulse reponse of the complete system.

$h(t) * \delta(t) =$

$h(t) * \delta(t+1) =$



$h_{cas}(t) =$