GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

Final Exam

DATE: 29-July-03 COURSE: ECE 2025

NAME: Key (LAST, FIRST) STUDENT #: ____________

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-10:00am (D. Taylor)

L02:Tues-12:00am (T. Michaels)

L03:Tues-2:00pm (D. Taylor)

L04:Tues-4:00pm (D. Taylor)

L06:Mon-4:00pm (T. Michaels)

- Write your name on the front page ONLY. Remove the two Magic Sheets of Fourier and z-transform tables and properties from the back of the exam for your reference, but **DO NOT UNSTAPLE** the rest of the exam.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive any partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	5	
2	5	
3	20	
4	20	
5	20	
6	20	

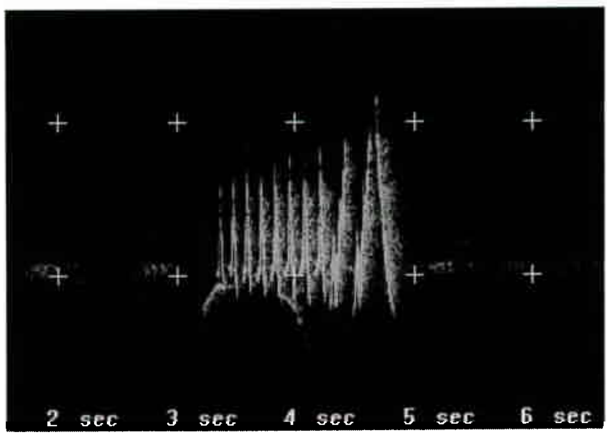
Problem	*Value*	*Score*
7	20	
8	20	
9	20	
TOTAL	150	

Problem Su03-F.1:
The following two questions are to be entered on the cover sheet of your exam.

(a) What is your name?

(b) What is your recitation section? Circle the day and time when your recitation section meets on the cover sheet of this exam.

Problem Su03-F.2:
Shown in the figure below is the spectrogram of a signal.



Which of the following signals does this spectrogram most likely correspond to? Circle one and only one answer.

(a) Female speech

(b) A model airplane

(c) A creaking door

(d) A chestnut warbler (a type of bird)

(e) A singing voice

Problem Su03-F.3:

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Evaluate the following expression: $|(1+3j)e^{j(0.8t)}|^2$.

$$\left|(1+3j)\,e^{j(0.8t)}\right|^2 = |1+3j|^2 = 10$$

(b) Evaluate the following derivative: $\frac{d}{dt}\Big[\sin(\pi t - \pi/2)u(t-1)\Big]$

$$\frac{d}{dt}\left[\sin(\pi t - \pi/2)\,u(t-1)\right] = \pi\cos(\pi t - \pi/2)\,u(t-1) + \sin(\pi t - \pi/2)\,\delta(t-1)$$

$$= \pi\cos(\pi t - \pi/2)\,u(t-1) + \delta(t-1)$$

(c) Evaluate the following integral: $\int_{-\infty}^{\infty} \sin(200\pi\tau + \pi/2)\delta(\tau - 1)e^{-j\pi\tau}d\tau$

$$\int_{-\infty}^{\infty} \sin(200\pi\tau + \pi/2)\,\delta(\tau-1)\,e^{-j\pi\tau}\,d\tau = \int_{-\infty}^{\infty} \sin(200\pi + \pi/2)\,e^{-j\pi}\,\delta(\tau-1)\,d\tau$$

$$= \underbrace{\sin(200\pi + \pi/2)\,e^{-j\pi}}_{-1}\ \underbrace{\int_{-\infty}^{\infty} \delta(\tau-1)\,d\tau}_{1} = -1$$

(d) Evaluate the following integral: $\int_{t-1}^{t+2} e^{-3\tau}\delta(\tau-5)d\tau$

$$\int_{t-1}^{t+2} e^{-3\tau}\delta(\tau-5)d\tau = e^{-15}\int_{t-1}^{t+2}\delta(\tau-5)d\tau = \begin{cases} e^{-15} & 3 \le t \le 6 \\ 0 & \text{else} \end{cases}$$

(e) Evaluate the following convolution: $\left\{e^{3jt}\right\} * \left\{e^{-5t}u(t)\right\}$

$$e^{-5t}u(t) \longleftrightarrow \frac{1}{5+j\omega}$$

So

$$e^{3jt} * e^{-5t}u(t) = \left[\frac{1}{5+j\omega}\right]_{\omega=3} e^{3jt} = \frac{1}{5+3j}e^{3jt}$$

(f) Evaluate the following convolution: $\delta(t-2) * \delta(t+1)$

$$\delta(t-2) * \delta(t+1) = \delta(t-1)$$

Problem Su03-F.4:

Which of the following signals are periodic? For those that are periodic, find the fundamental period, T_0, and the Fourier Series coefficients a_k for all k.

(a) $x(t) = 3 + \sin(199\pi t - \pi/2) + 2\cos(200\pi t - \pi/5)$

Periodic? Yes or **No**. (Circle answer).

If YES:

$\omega_0 = 2\pi f_0 = \pi \Rightarrow f_0 = 1/2$

$T_0 = 2$

$\boxed{T_0 = 2}$

$a_k =$

3	$k=0$
$\frac{1}{2j} e^{-j\pi/2} = -\frac{1}{2}$	$k = \pm 199$
$e^{-j\pi/5}$	$k = 200$
$e^{j\pi/5}$	$k = -200$

(b) $x(t) = 3\cos(225\pi t + \pi/4)\cos(2000\pi t) = \frac{3}{2}\left[\cos(2225\pi t + \pi/4) + \cos(1775\pi t - \pi/4)\right]$

Periodic? Yes or **No**. (Circle answer).

If YES:

$\omega_0 = 25\pi \Rightarrow f_0 = 25/2$

$T_0 = 2/25 = 0.08$

$\boxed{T_0 = 0.08}$

$a_k =$

$\frac{3}{4} e^{j\pi/4}$	$k = 89$
$\frac{3}{4} e^{-j\pi/4}$	$k = -89$
$\frac{3}{4} e^{-j\pi/4}$	$k = 71$
$\frac{3}{4} e^{j\pi/4}$	$k = -71$

(c) $x(t) = \cos(\sqrt{2}\pi t + \pi/5) + \cos(\pi t - \pi/5)$

Periodic? **Yes or No.** (Circle answer).

If YES:

$T_0 =$

$a_k =$

(d) $x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{|k|+1} e^{-j30k\pi t}$

$\omega_0 = 2\pi f_0 = 30\pi \Rightarrow f_0 = 15$

$T_0 = 1/15$

Periodic? **Yes or No**. (Circle answer).

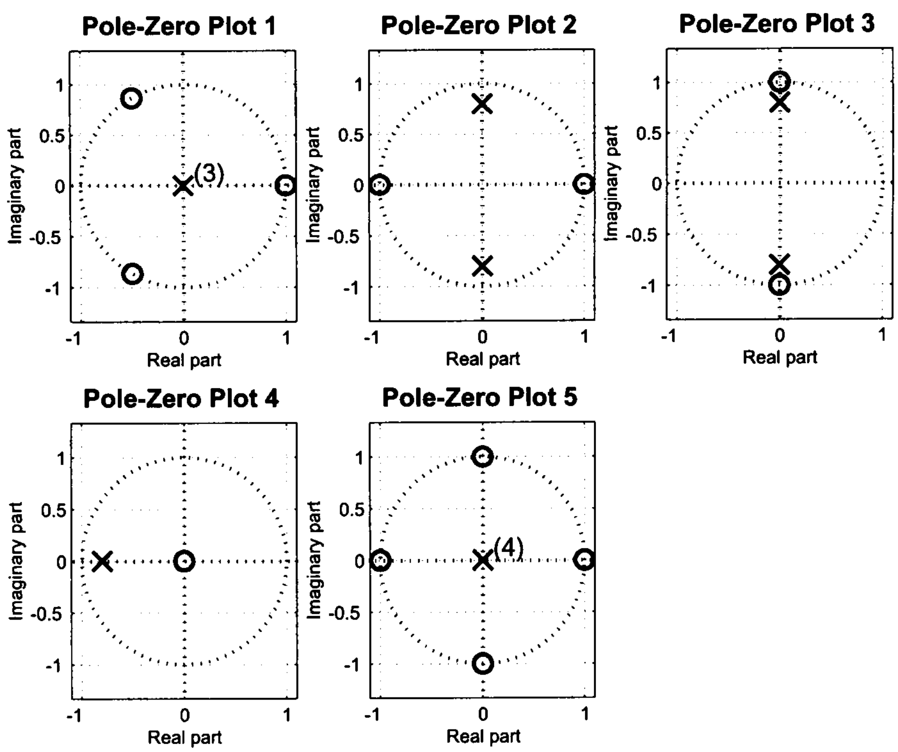
If YES:

$T_0 = 1/15$

$a_k = \frac{1}{\lvert k \rvert + 1}$

Problem Su03-F.5:

Shown in the figure below are the pole-zero diagrams for the system functions of five different linear time-invariant, discrete time systems. Assume that the poles in plots 2, 3, and 4 have a magnitude of 0.8, i.e., lie at a distance of 0.8 from the origin, and that the angle of the two complex zeros in plot 1 are at $\pm 2\pi/3$ radians.



(a) Assume that each of the systems represented by the above pole-zero plots has impulse response $h_k[n]$, where k is the index shown in the title of the pole-zero plot. Determine whether each system is FIR or IIR by circling your answer below.

(a) System 1: **FIR** or **IIR** (Circle one)

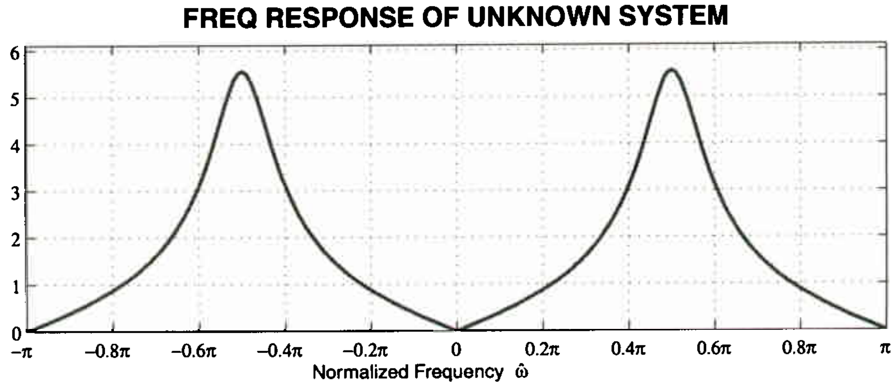
(b) System 2: **FIR** or **IIR** (Circle one)

(c) System 3: **FIR** or **IIR** (Circle one)

(d) System 4: **FIR** or **IIR** (Circle one)

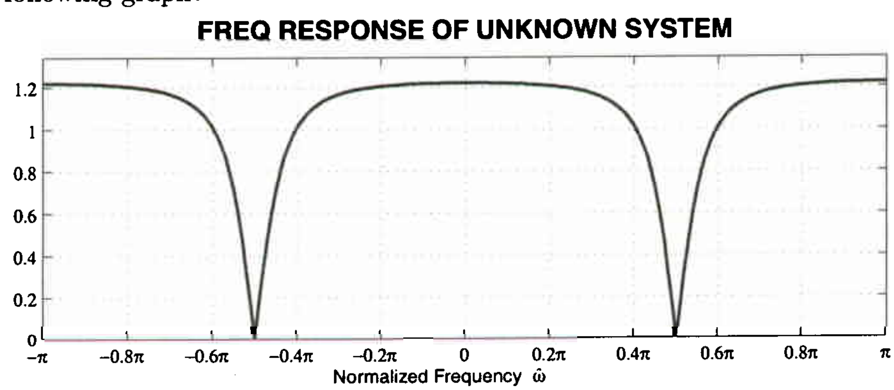
(e) System 5: **FIR** or **IIR** (Circle one)

(b) Which of the above pole-zero plots represents the system whose frequency response is given in the following graph?



System Number = 2

(c) Which of the above pole-zero plots represents the system whose frequency response is given in the following graph?



System Number = 3

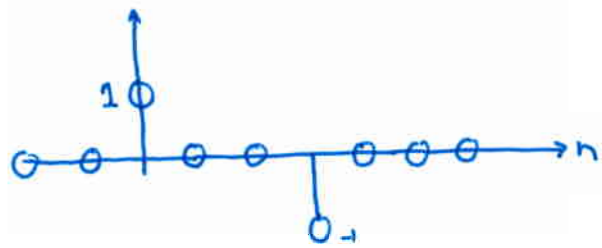
(d) Find the unit sample response for the system corresponding to **Pole-Zero Plot 1**, and make a carefully labeled plot of $h_1[n]$ versus n.

$$H_1(z) = (1 - z^{-1})(1 - e^{-j2\pi/3} z^{-1})(1 - e^{j2\pi/3} z^{-1})$$

$$= (1 - z^{-1})(1 - 2\cos(2\pi/3) z^{-1} + z^{-2})$$

$$= (1 - z^{-1})(1 + z^{-1} + z^{-2}) = 1 - z^{-3}$$

$$\Rightarrow h[n] = \delta(n) - \delta[n-3]$$



(e) Which system is characterized by a difference equation of the form

$$y[n] = -0.64y[n-2] + x[n] + x[n-2]$$

where $x[n]$ is the input and $y[n]$ is the output?

System Number = 3

(f) Which of the above pole-zero plots represents a system for which

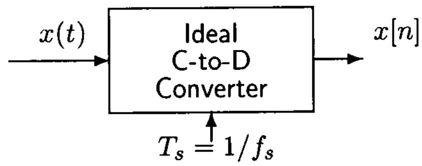
$$h_k[n] * \left[\cos(n\pi/2)\right] = 0$$

If none, write **NONE**.

System Number(s) = 3 & 5

Problem Su03-F.6:

An ideal continuous-to-discrete converter is shown in the following figure.



(a) The input to the ideal C-D converter is

$$x(t) = 11\cos(\omega t + \pi/8)$$

and the output signal is

$$x[n] = 11\cos(0.4\pi n + \pi/8)$$

If sample rate is $f_s = 2000$ samples/sec, find two possible frequencies, ω, for the input signal $x(t)$.

$\omega T \longrightarrow \omega n T_s = 0.4\pi n$, so $\omega = 0.4\pi / T_s = 800\pi$

We can add integer multiples of ω_s, so

$\omega = 800\pi \pm 4000\pi k = \ldots, -3200\pi, 800\pi, 4800\pi, \ldots$

(b) If the input to the ideal C-D converter is

$$x(t) = 11\cos(6000\pi t + \pi/8)$$

and the output signal $x[n]$ is

$$x[n] = 11\cos(0.4\pi n + \pi/8)$$

find two possible sampling frequencies, f_s, for the ideal C-D converter.

$6000\pi t \longrightarrow 6000\pi n T_s = 0.4\pi n \Rightarrow f_s = \frac{1}{T_s} = \frac{6000}{0.4} = 15{,}000$ Hz
(No aliasing)

We can also get the given x[n] by aliasing.

$6000\pi t \longrightarrow 6000\pi n T_s = (0.4\pi + 2\pi) n \Rightarrow f_s = \frac{6000}{2.4\pi} = 2500$ Hz

(c) If the input to the ideal C-D converter is

$$x(t) = 11\cos(6000\pi t + \pi/8)$$

and the sampling rate is $f_s = 4000$, find the output $x[n]$.

$$x[n] = 11 \cdot \cos\left(6000\pi n/4000 + \pi/8\right)$$

$$= 11 \cdot \cos\left(\frac{3\pi}{2} n + \frac{\pi}{8}\right) = 11 \cdot \cos\left(\frac{\pi}{2} n - \frac{\pi}{8}\right)$$

(d) If the input to the ideal C-D converter is

$$x(t) = 3\cos(2000\pi t) \cdot \sin(3000\pi t + \pi/3)$$

what is the Nyquist rate for sampling $x(t)$?

max frequency $= \omega_{max} = 2000\pi + 3000\pi = 5000\pi$

$\Rightarrow f_{max} = 2500$ Hz

So, $f_{NYQUIST} = 2 \cdot f_{max} = 5000$ Hz

Problem Su03-F.7:

The four parts of this problem are independent of one another. Brute force attempts at these questions will probably lead you nowhere. Be clever! **Hint:** On one of the parts, it will help to remember the rule from your freshman calculus class about computing the derivative of a quotient:

$$\frac{d}{dx}\left\{\frac{f}{g}\right\} = \frac{g\frac{d}{dx}\{f\} - f\frac{d}{dx}\{g\}}{g^2}$$

(a) Compute this integral (simplify as much as possible):

$$\int_{-\infty}^{\infty} \frac{e^{j\omega 3}}{7+j\omega}\, d\omega = 2\pi \overbrace{\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{j\omega t}}{7+j\omega}\, d\omega\right]}^{\text{inverse Fourier transform!}}\Bigg|_{t=3}$$

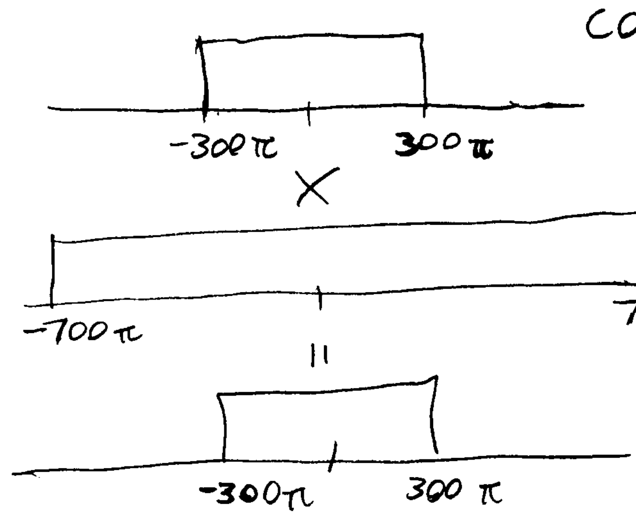
$$= 2\pi e^{-7t} u(t)\Big|_{t=3} = \boxed{2\pi e^{-21}}$$

(b) Compute this integral (simplify as much as possible):

$$\underbrace{\frac{1}{\pi^2}\int_{-\infty}^{\infty} \frac{\sin(300\pi\tau)\sin(700\pi(t-\tau))}{\tau(t-\tau)}\, d\tau}_{\text{convolution in time corresponds to multiplication in frequency}} = \boxed{\frac{\sin(300\pi t)}{\pi t}}$$

$$\mathcal{F}\left\{\frac{\sin(300\pi t)}{\pi t}\right\} =$$

$$\mathcal{F}\left\{\frac{\sin(700\pi t)}{\pi t}\right\} =$$



(c) The "ramp filter" is often used in medical imaging applications such as X-ray computer-aided tomography. It has a frequency response given by

$$H(j\omega) = \begin{cases} j\omega & \text{for } |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

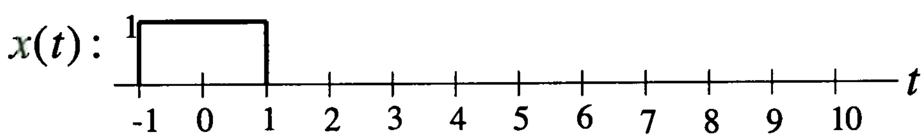
Find the impulse response, $h(t)$, of this filter.

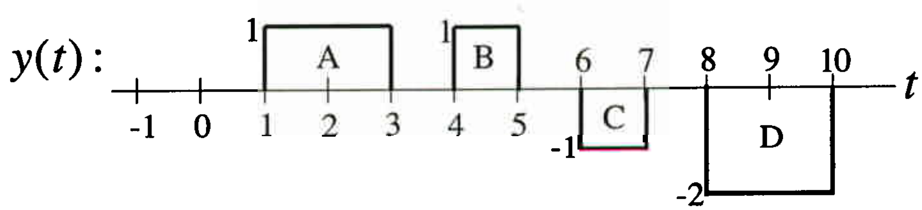
$$x(t) = \frac{d}{dt}\left\{\frac{\sin(w_0 t)}{\pi t}\right\}$$

$$= \frac{\pi t \times w_0 \sin(w_0 t) - \pi \sin(w_0 t)}{\pi^2 t^2}$$

$$= \boxed{\frac{w_0 t \sin(w_0 t) - \sin(w_0 t)}{\pi t^2}}$$

(d) Suppose a signal $x(t)$ is input to a linear LTI system with impulse response $h(t)$. The input $x(t)$ and the output $y(t) = (x * h)(t)$ are given by:



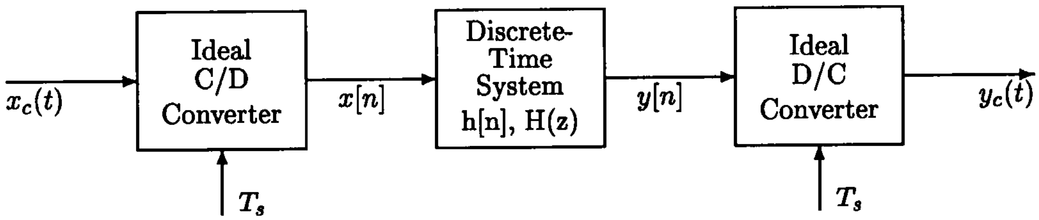


What is $h(t)$? (Hint: Your answer should contain four terms. If you are totally stuck, concentrate on what terms you need to get the blocks labeled "A" and "D" first, then think about blocks "B" and "C".)

$$h(t) = \delta(t-2) + \delta(t-5) - \delta(t-6) - 2\delta(t-9)$$

Problem Su03-F.8:

This problem will explore the following system:



$$x[n] = x_c(nT_s) \qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T_s}(t - nT_s)}{\frac{\pi}{T_s}(t - nT_s)}$$

The LTI discrete-time system in the middle has the system function

$$H(z) = \frac{1}{1 - \frac{1}{\sqrt{3}} z^{-1}}$$

(a) What is the impulse response $h[n]$ of the discrete-time system?

From table,

$$h[n] = \left(\frac{1}{\sqrt{3}}\right)^n u[n]$$

(b) What is the difference equation that specifies the discrete-time system in terms of the relation between $y[n]$ and $x[n]$?

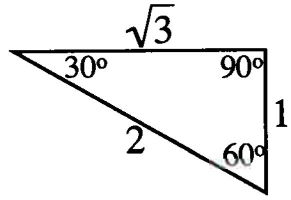
$$Y(z)\left[1 - \frac{1}{\sqrt{3}} z^{-1}\right] = X(z)$$

$$y[n] - \frac{1}{\sqrt{3}} y[n-1] = x[n]$$

$$y[n] = \frac{1}{\sqrt{3}} y[n-1] + x[n]$$

(c) Suppose $x_c(t) = 2\cos(50\pi t)$ and $T_s = 0.01$. What is $y_c(t)$?

Hint: Remember the 30°-60°-90° triangle from your high school geometry class:



$$\omega T_s = 50\pi \times 0.01 = 0.5\pi$$

So no aliasing

$$H(e^{j\frac{\pi}{2}}) = \frac{1}{1-\frac{1}{\sqrt{3}}e^{-j\frac{\pi}{2}}} = \frac{1}{1+\frac{1}{\sqrt{3}}j} = \frac{\sqrt{3}}{\sqrt{3}+j}$$

$$= \frac{\sqrt{3}}{2e^{j\pi/6}} = \frac{\sqrt{3}}{2}e^{-j\pi/6}$$

$$y_c(t) = 2\cdot\frac{\sqrt{3}}{2}\cos(50\pi t - \pi/6) = \boxed{\sqrt{3}\cos(50\pi t - \pi/6)}$$

could also work the complex number by:

$$\frac{1}{(1+\frac{1}{\sqrt{3}}j)}\frac{1-\frac{1}{\sqrt{3}}j}{(1-\frac{1}{\sqrt{3}}j)} = \frac{1-\frac{1}{\sqrt{3}}j}{1+\frac{1}{3}} = \frac{\frac{1}{\sqrt{3}}(\sqrt{3}-j)}{\frac{4}{3}} = \frac{\frac{2}{\sqrt{3}}e^{-j\frac{\pi}{6}}}{\frac{4}{3}} = \frac{\sqrt{3}}{2}e^{-j\pi/6}$$

(d) Now suppose $x_c(t) = (\sqrt{3}-1)\cos(200\pi t)$ and $T_s = 0.01$. What is $y_c(t)$?

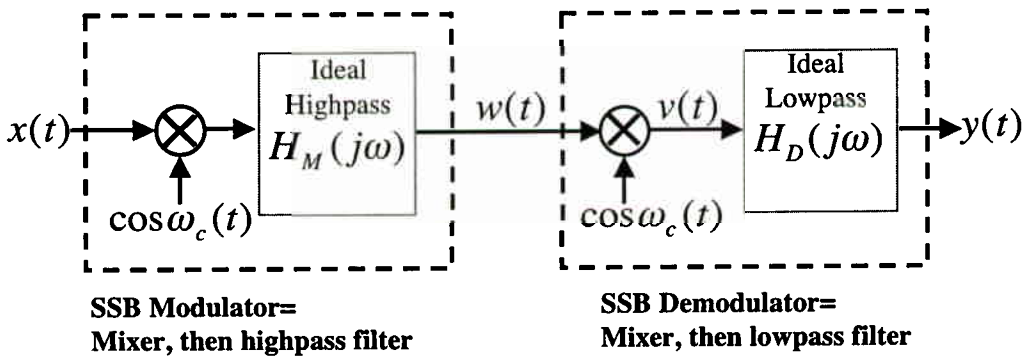
$$\omega T_s = 200\pi \times 0.01 = 2\pi \rightarrow \text{Aliases to DC!}$$

$$H(e^{j0}) = \frac{1}{1-\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{3}-1}$$

$$y_c(t) = (\sqrt{3}-1)\times\frac{\sqrt{3}}{\sqrt{3}-1} = \boxed{\sqrt{3}}$$

Problem Su03-F.9:

In lecture and in lab, we explored a communication strategy called *double-sideband amplitude modulation*. In Problem Set #11, there was a problem on *quadrature modulation*. In this problem, we will explore another strategy called *single-sideband modulation* (SSB). Block diagrams of an SSB Modulator and an SSB Demodulator are given below.



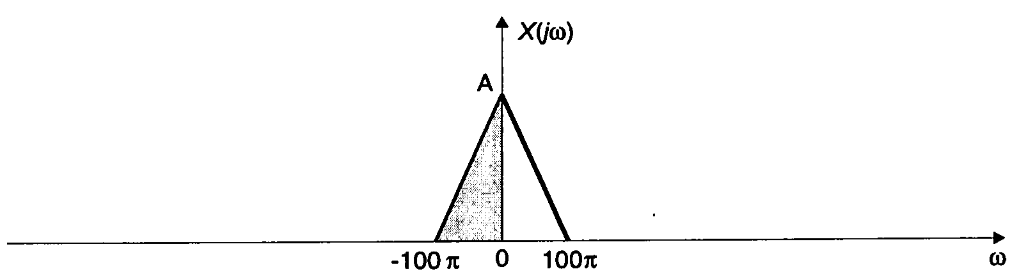
H_M is a *highpass filter*, here specified by:

$$H_M(j\omega) = \begin{cases} 1 & \text{for } |\omega| \geq 250\pi \\ 0 & \text{otherwise} \end{cases}$$

H_D is a *lowpass* filter, here specified by:

$$H_D(j\omega) = B[u(\omega + 100\pi) - u(\omega - 100\pi)]$$

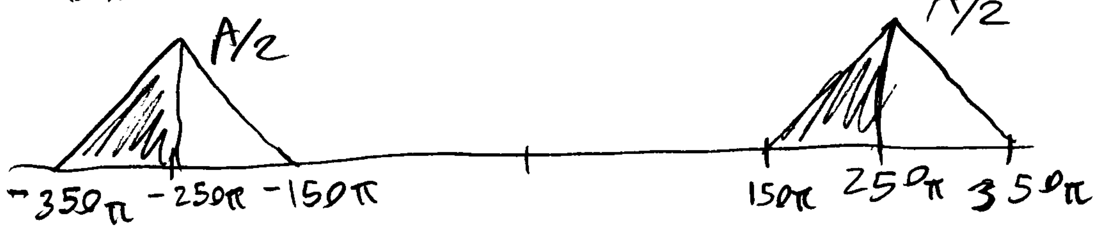
Notice this looks very similar to the double-sideband AM communication system you explored in lab; the only change is the addition of the highpass filter H_M to the modulator. Suppose that $\omega_c = 250\pi$, and the input signal $x(t)$ has this "typical" Fourier transform $X(j\omega)$:

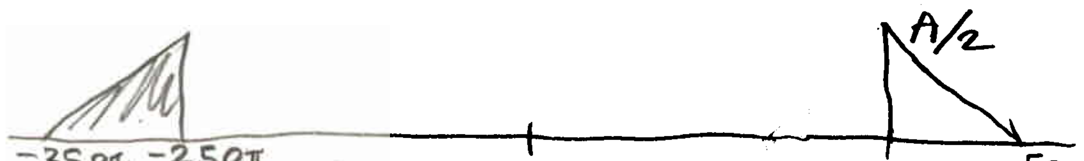


We will now trace the input signal through the modulation and demodulation stages to the output by analyzing it in the Fourier domain. In your sketches, be sure to **label the amplitudes and correctly note what parts are "shaded" and what parts are "unshaded."** (Keeping careful track of the amplitudes will be important to part (c).) You will find it easiest to think *graphically*; you will not need to write any complicated equations.

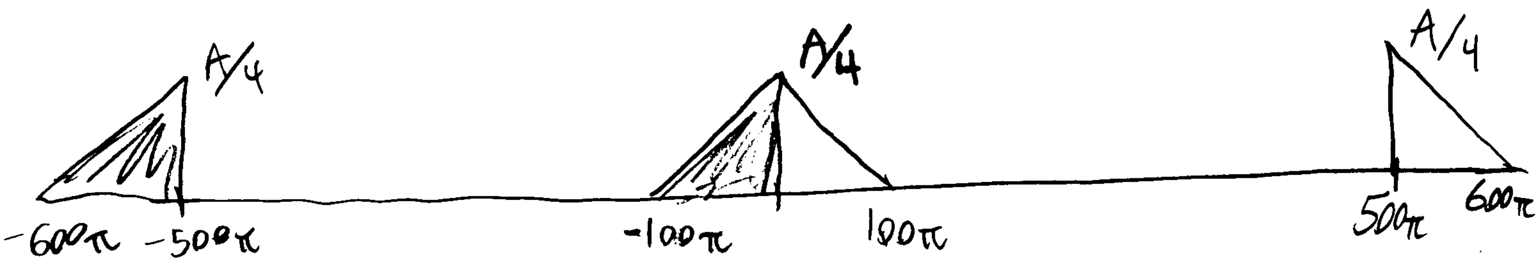
After first mixer:

(a) Sketch $W(j\omega)$, the Fourier transform of $w(t)$.



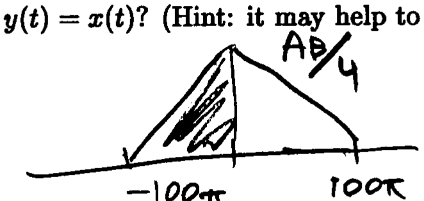


(b) Sketch $V(j\omega)$, the Fourier transform of $v(t)$.



(c) How should B, the gain of the lowpass filter in the demodulator, be chosen so that the input exactly equals the ~~input~~ output, i.e. $y(t) = x(t)$? (Hint: it may help to sketch $Y(j\omega)$.)

$Y(j\omega) =$



need $\boxed{B = 4}$

(d) Which technique – double-sideband AM or single-sideband modulation – do you think makes more efficient use of the spectrum? **Briefly** explain your reasoning.

$\boxed{\text{SSB}}$ - it uses half the bandwidth of DSB