

Sol.

①

12.1

$$(a) f_{\max} = \frac{\delta \omega \pi}{2\pi} = 4000$$

$$\omega_s = \frac{2\pi}{T_s} = 2\pi \cdot 2f_{\max} = 2\pi \cdot 8000$$

$$(b) H(e^{j\omega T_s}) = H(e^{\frac{j\omega}{f_s}})$$
$$= 3e^{-j4\omega/8000}$$
$$= 3e^{-j0.0005\omega}$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}), & |\omega| < 2\pi \cdot 4000 \\ 0, & \text{else} \end{cases}$$

$$Y_c(j\omega) = H_{\text{eff}}(j\omega) X_c(j\omega) = 3e^{-j0.0005\omega} \text{ for } |\omega| < 8000\pi$$

$$y_c(t) = 3x_c(t - 0.0005)$$

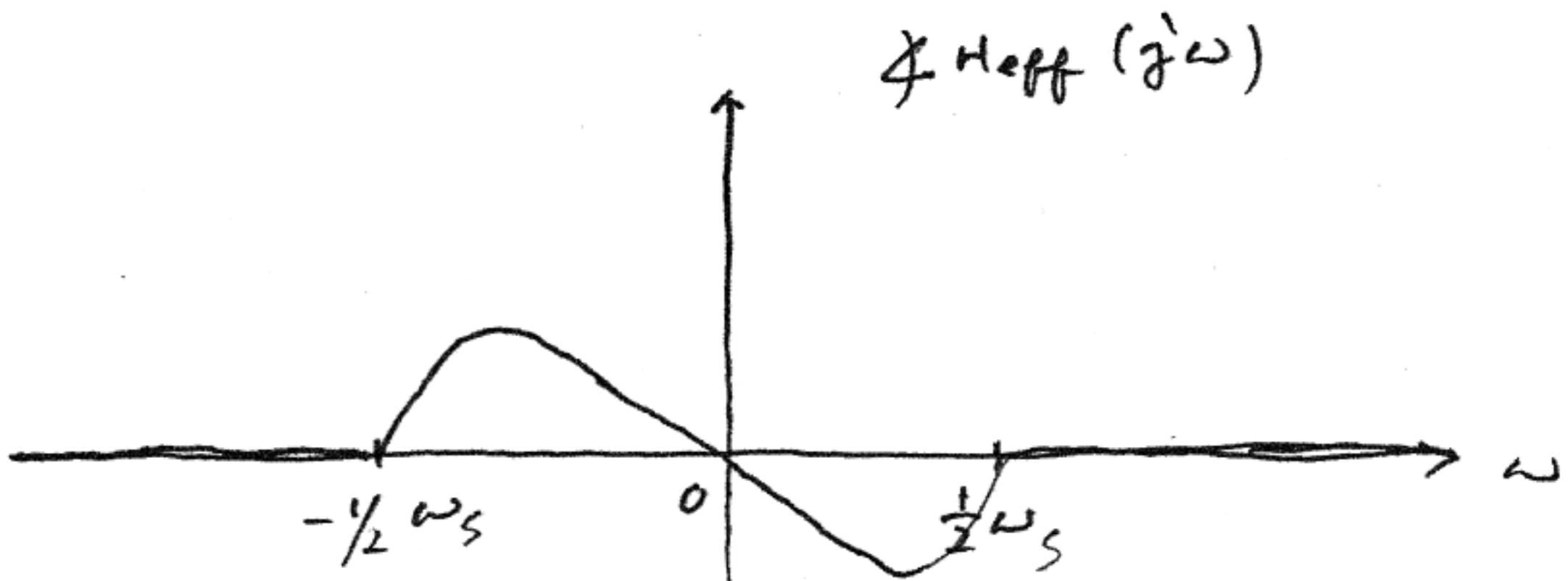
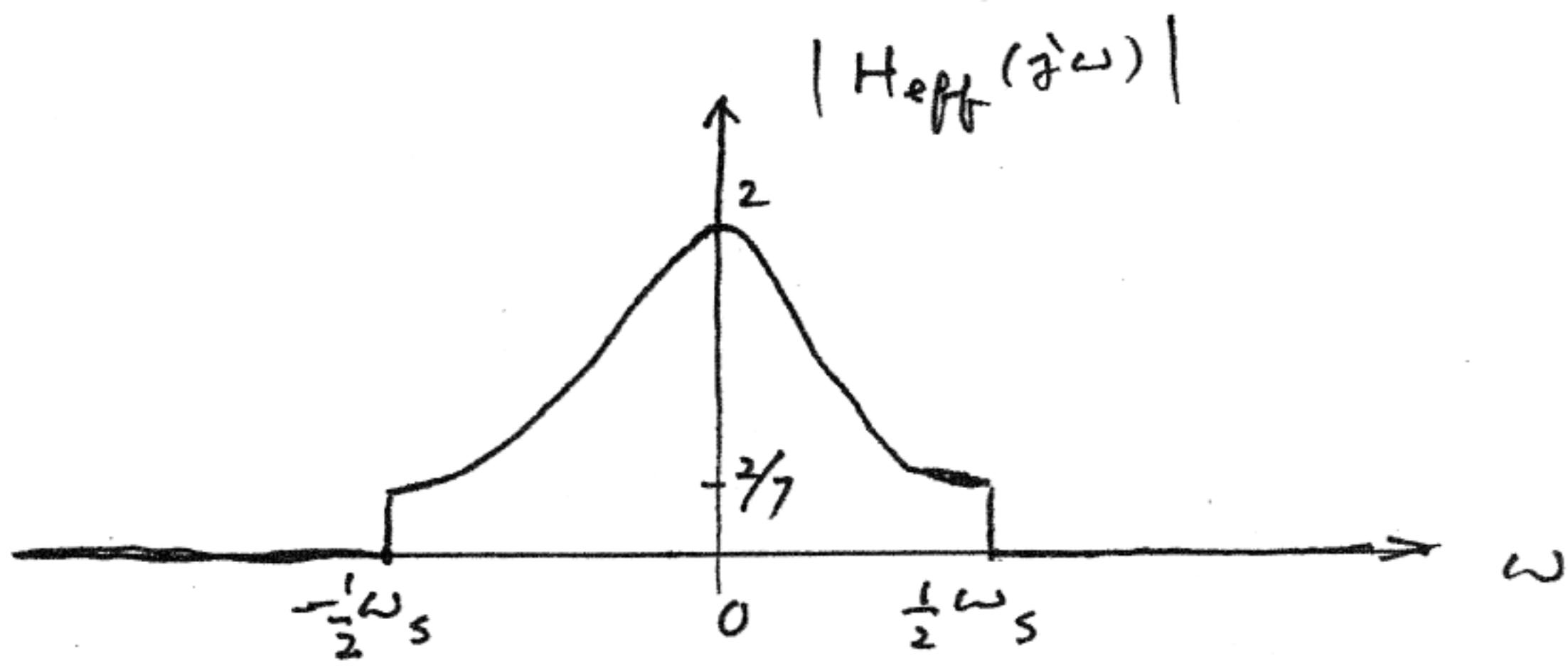
$$(c) H(e^{j\omega T_s}) = \frac{0.5}{1 - 0.75e^{-j\omega/8000}}$$

$$H_{\text{eff}}(j\omega) = \begin{cases} \frac{0.5}{1 - 0.75e^{-j\omega/8000}}, & |\omega| \leq 2\pi \cdot 4000 \\ 0, & \text{else} \end{cases}$$

$$= |H_{\text{eff}}(j\omega)| e^{j \varphi_{H_{\text{eff}}}(j\omega)} \quad (2)$$

$$|H_{\text{eff}}(j\omega)| = \begin{cases} \frac{e^{-5}}{\sqrt{\frac{25}{16} - \frac{3}{2} \cos \frac{\omega}{8000}}} & , |\omega| \leq 2\pi \cdot 4000 \\ 0, & \text{else} \end{cases}$$

$$\varphi_{H_{\text{eff}}}(j\omega) = \begin{cases} -\arctan \left(\frac{0.75 \sin \frac{\omega}{8000}}{1.75 \cos \frac{\omega}{8000}} \right) & , |\omega| \leq 2\pi \cdot 4000 \\ 0 & , \text{else} \end{cases}$$



$$\omega_s = 2\pi \cdot 8000$$

12.2 Sol. :

(3)

$$(a) \quad X_a(z) = \frac{1}{2} \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \frac{1}{2} \sum_{n=0}^{+\infty} \left(-\frac{1}{2}z^{-1}\right)^n$$

$$= \frac{1}{2} \cdot \frac{1}{1 - (-\frac{1}{2})z^{-1}} = \frac{\frac{1}{2}}{1 + \frac{1}{2}z^{-1}}$$

$$\left(x[n] = b_0 a_1^n u[n] \xrightarrow{\text{Z transform}} \frac{b_0}{1 - a_1 z^{-1}} \right)$$

$$(b) \quad 3\left(\frac{1}{3}\right)^n u[n] \xrightarrow{\text{Z transform}} \frac{3}{1 - \frac{1}{3}z^{-1}}$$

$$u(n+1) \xrightarrow{\text{Z transform}} z^{-1} \cdot \frac{1}{1 - z^{-1}}$$

$$X_b(z) = \frac{3}{1 - \frac{1}{3}z^{-1}} - \frac{2z^{-1}}{1 - z^{-1}}$$

$$= \frac{3 - 3z^{-1} - 2z^{-1} + \frac{2}{3}z^{-2}}{(1 - \frac{1}{3}z^{-1})(1 - z^{-1})}$$

$$= \frac{3 - 5z^{-1} + \frac{2}{3}z^{-2}}{1 - \frac{4}{3}z^{-1} + \frac{2}{3}z^{-2}}$$

$$(c) \quad \delta[n] \xrightarrow{\text{Z-trans.}} 1$$

$$u[n] \xrightarrow{\text{Z-trans.}} \frac{1}{1 - z^{-1}}$$

$$u[n-1] \xrightarrow{\text{Z-trans.}} z^{-1} \cdot \frac{1}{1 - z^{-1}}$$

$$X_c(z) = -1 + \frac{z^{-1}}{1 - z^{-1}}$$

$$= \frac{-1 + 2z^{-1}}{1 - z^{-1}}$$

12.3

(4)

$$(a) H_a(z) = \frac{1}{1-0.5z^{-1}} + \frac{z^{-1}}{1-0.5z^{-1}}$$

$$\frac{1}{1-0.5z^{-1}} \xrightarrow{\text{Z}^+ \text{ transform}} (0.5)^n u(n)$$

$$\frac{z^{-1}}{1-0.5z^{-1}} \xrightarrow{\text{Z}^- \text{ trans.}} (0.5)^{n-1} u(n-1)$$

(the factor z^+ at the numerator corresponds to a delay in time)

$$h_a(n) = (0.5)^n u(n) + (0.5)^{n-1} u(n-1)$$

$$(b) H_b(z) = \frac{0.5}{1+0.75e^{j0.3\pi} z^{-1}} + \frac{0.5}{1-0.75e^{-j0.3\pi} z^{-1}}$$

since $b_0 a_i^n u(n) \xrightarrow{\text{Z transform}} \frac{b_0}{1-a_i z^{-1}}$ where a_i can be complex,
(and Z transform is \mapsto),

$$\frac{0.5}{1+0.75e^{j0.3\pi} z^{-1}} \xrightarrow{\text{Z}^+ \text{ transform}} 0.5 (-0.75e^{j0.3\pi})^n u[n]$$

$$\frac{0.5}{1-0.75e^{-j0.3\pi} z^{-1}} \xrightarrow{\text{Z}^+ \text{ transform}} 0.5 (0.75e^{-j0.3\pi})^n u[n]$$

(5)

$$h_b[n] = \frac{1}{2} \left[\left(-\frac{3}{4}\right)^n e^{j \cdot 3\pi n} + \left(\frac{3}{4}\right)^n e^{-j \cdot 3\pi n} \right] u(n)$$

$$= \begin{cases} \left(\frac{3}{4}\right)^n \cos(3\pi n) u(n) & \text{for } n \text{ even}, \\ \left(\frac{3}{4}\right)^n (-j) \sin(3\pi n) u(n) & \text{for } n \text{ odd} \end{cases}$$

$$(c) H_c(z) = \frac{0.6}{1+0.6z^{-1}} + \frac{z^{-1}}{1+0.6z^{-1}}$$

$$\frac{0.6}{1+0.6z^{-1}} \xrightarrow{z^{-1} \text{ transform}} 0.6 [-0.6]^n u[n]$$

$$\frac{z^{-1}}{1+0.6z^{-1}} \xrightarrow{z^{-1} \text{ transform}} [-0.6]^{n-1} u[n-1]$$

$$h_c(n) = 0.6 [-0.6]^n u[n] + [-0.6]^{n-1} u[n-1]$$

$$= 0.6 \delta[n] + 0.64 [-0.6]^{n-1} u[n-1]$$

(6)

12. 4.

$$S_1 : Y(z) = 0.4z^{-1}Y(z) + X(z) + z^{-1}X(z)$$

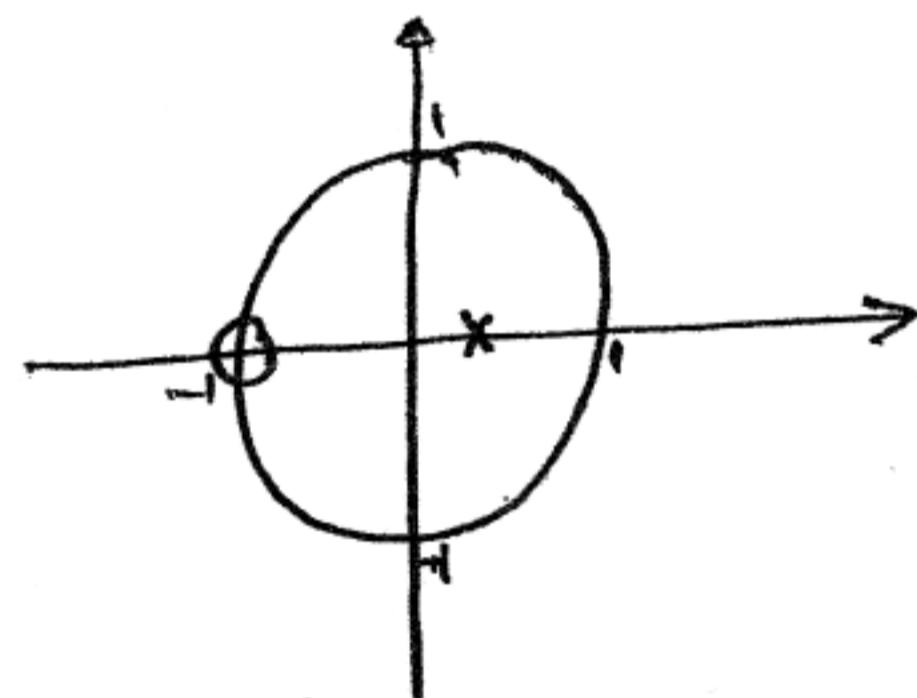
$$Y(z)[1 - 0.4z^{-1}] = X(z)[1 + z^{-1}]$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{1 + z^{-1}}{1 - 0.4z^{-1}} = \frac{N(z)}{D(z)}$$

Zero : Let $N(z) = 0$, $\Rightarrow 1 + z^{-1} = 0$ $z_0 = -1$

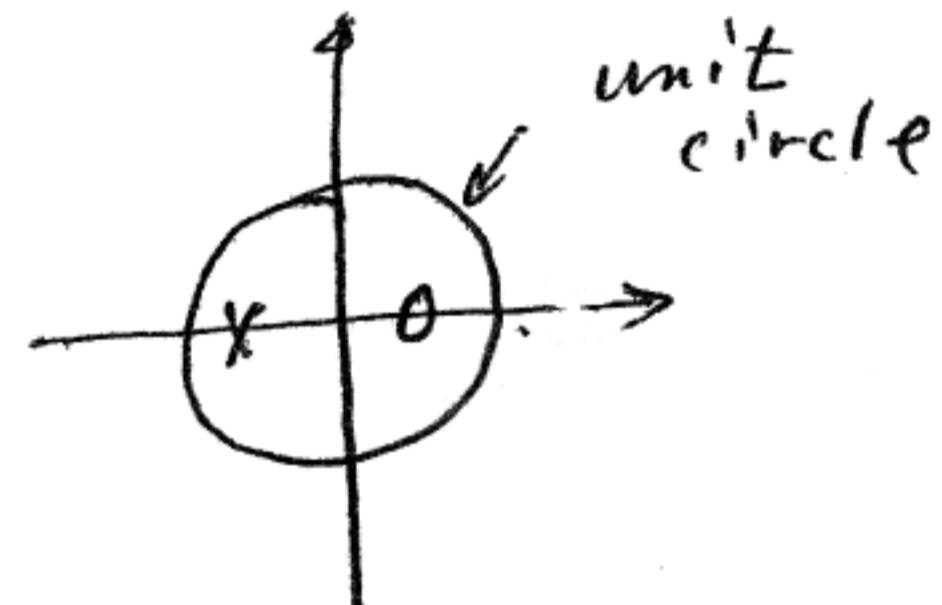
Pole : Let $D(z) = 0$, $\Rightarrow 1 - 0.4z^{-1} = 0$, $z_p = 4$



$$S_2 : H(z) = \frac{1.5 - z^{-1}}{1 + 0.75z^{-1}}$$

$$\text{Zero : } z_0 = \frac{1}{1.5} = \frac{2}{3}$$

$$\text{Pole : } z_p = -0.75$$

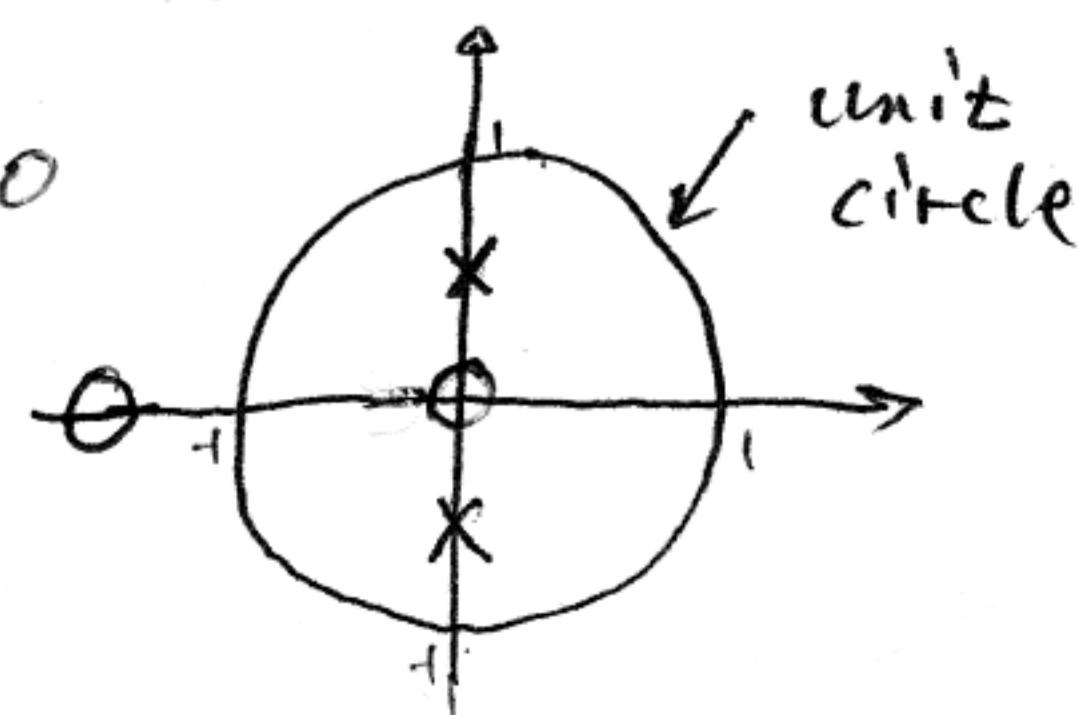


$$S_3 : H(z) = \frac{1 + 2.0z^{-1}}{1 + 0.5z^{-2}},$$

$$\text{Zero : } z_{01} = -2.0, z_{02} = 0$$

$$\text{Two poles : } z_{p1} = -j\sqrt{5}$$

$$z_{p2} = j\sqrt{5}$$



(7)

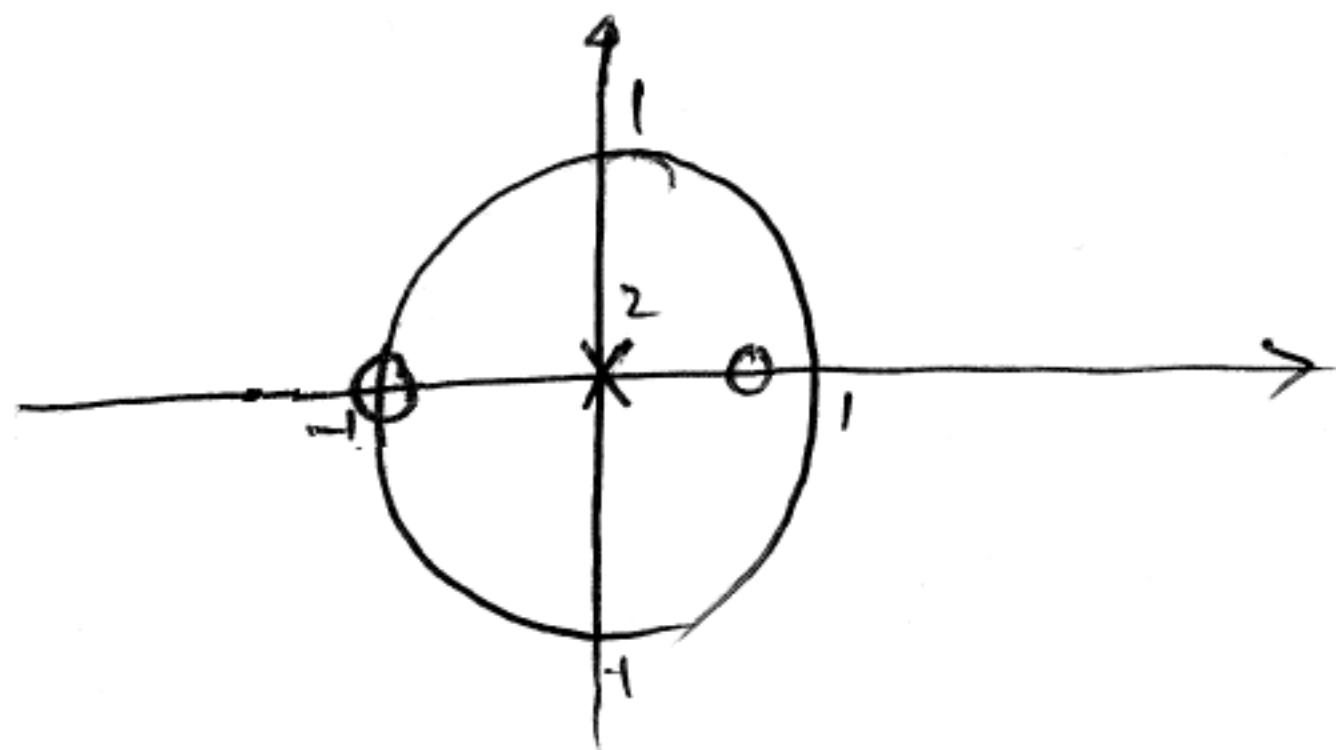
$$S_4 : H(z) = 1 + \frac{1}{4}z^{-1} + \left(-\frac{3}{4}\right)z^{-2}$$

$$= \frac{z^2 + \frac{z}{4} - \frac{3}{4}}{z^2}$$

$$= \frac{(z+1)(z-\frac{3}{4})}{z^2}$$

Two zeroes : $z_{o1} = \frac{3}{4}$, $z_{o2} = -1$

Two poles : $z_{p1} = z_{p2} = 0$
 (or one 2nd order pole)



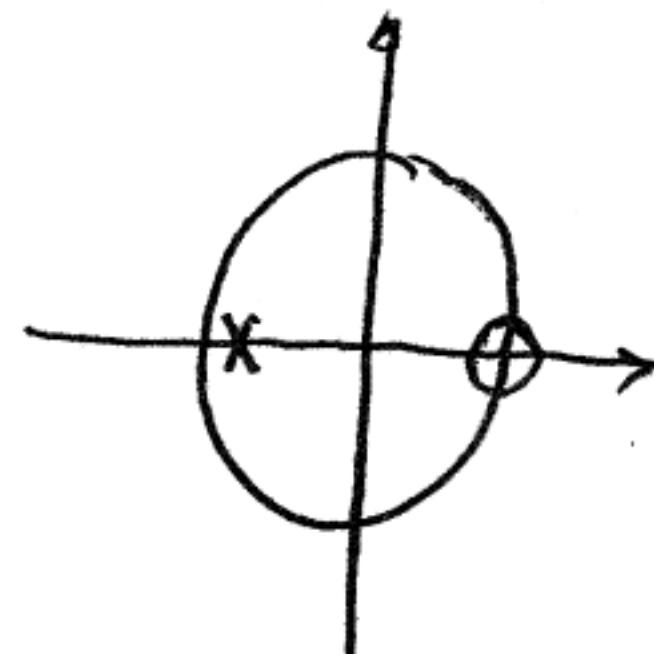
Observations :

FIR filters have poles only at the origin
 IIR " can " " elsewhere

$$(a) H(z) = \frac{1-z^{-1}}{1+0.9z^{-1}}$$

$$\text{Zero: } z_0 = 1$$

$$\text{Pole: } z_p = -0.9$$



$$(b) H(z) = \frac{1}{1+0.9z^{-1}} \div \frac{z^{-1}}{1+0.9z^{-1}}$$

$$h[n] = (-0.9)^n u[n] - [-0.9]^{n-1} u[n-1]$$

$$= \delta[n] + 1.9(-0.9)^{n-1} u(n-1)$$

(c) $(-0.9)^{n-1} \xrightarrow{n \rightarrow +\infty} 0$, the feedback dies out for n large, and

$h[n] \xrightarrow{n \rightarrow +\infty} 0$ the system is stable
(or stable since z_p is within the unit circle.)

$$(d) H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}} = \frac{1-e^{-j\omega}}{1+0.9e^{-j\omega}}$$

$$H(e^{\frac{j\pi}{2}}) = \frac{1-e^{-j\frac{\pi}{2}}}{1+0.9e^{-j\frac{\pi}{2}}} = \frac{1+j}{1-0.9j} \\ \approx 1.05 e^{j0.48\pi}$$

$$y[n] = 2 \times 1.05 \cos(0.5\pi n + 0.48\pi)$$

$$= 2.1 \cos(0.5\pi n + 0.48\pi)$$

12. 6

$$(a) \text{ As } x[n] = \underbrace{2 \cos(0.8\pi n - \pi/3)}_{x_1[n]} + \underbrace{\cos(0.4\pi n - \pi)}_{x_2[n]} \quad (9)$$

consists of two cosine waves, it may be easier to use frequency response $H(e^{j\omega})$.

$$y[n] = -.9y[n-1] + x[n] - x[n-1]$$

$$Y(z) = -.9z^{-1}Y(z) + X(z) - z^{-1}X(z)$$

$$H(z) = \frac{1 - z^{-1}}{1 + .9z^{-1}}$$

then $H(e^{j\omega}) = H(z) \Big|_{\begin{array}{l} z = e^{j\omega} \\ -j\omega \end{array}}$

$$= \frac{1 - e^{-j\omega}}{1 + .9e^{-j\omega}}$$

$$H(e^{j0.8\pi}) = \frac{1 - e^{-j0.8\pi}}{1 + .9e^{-j0.8\pi}} + j0.45\pi$$

$$\therefore 3.19 e^{-j0.4\pi}$$

$$H(e^{j0.4\pi}) = \frac{1 - e^{-j0.4\pi}}{1 + .9e^{-j0.4\pi}}$$

$$\therefore .76 e^{j0.49\pi}$$

$$y[n] = 2 \times 3.19 \cos(.8\pi n - \pi/3 + .45\pi) + 1 \times .76 \cos(.4\pi n - \pi + .49\pi)$$

(10)

$$(b) x[n] = u[n] - u[n-6]$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{z^{-6}}{1-z^{-1}}$$

$$Y(z) = H(z) X(z)$$

$$= \frac{1-z^{-1}}{1+0.9z^{-1}} \cdot \frac{1-z^{-6}}{1-z^{-1}}$$

$$= \frac{1-z^{-6}}{1+0.9z^{-1}}$$

$$= \frac{1}{1+0.9z^{-1}} - \frac{z^{-6}}{1+0.9z^{-1}}$$

$$y[n] = [-0.9]^n u[n] - [-0.9]^{n-6} u[n-6]$$

$$(c) h[n] = \mathcal{Z}^{-1}[H(z)]$$

$$= [-0.9]^n u[n] - [0.9]^{n-1} u[n-1]$$

$$y[n] = h[n] - h[n-2]$$

$$= [0.9]^n u[n] - [-0.9]^{n-1} u[n-1] - \\ [-0.9]^{n-2} u[n-2] + [-0.9]^{n-3} u[n-3]$$

(d) use matlab. Truncate $h[n]$ to the significant value (when n is sufficiently large, the value of $h[n]$ is negligible).

Problem 12.7

Impulse J : FIR with non-zero values at $n=0, 1, 2, 3$

$$S_8, \quad H(z) = \frac{1}{3} (1-z^{-1})^3 = \frac{1}{3} - z^{-1} + z^{-2} - \frac{1}{3} z^{-3}$$

$$h[n] = \frac{1}{3} \delta[n] - \delta[n-1] + \delta[n-2] - \frac{1}{3} \delta[n-3]$$

Impulse K : IIR with a positive decaying factor since $n=1$

$$S_4, \quad H(z) = \frac{1-z^{-1}}{1-0.75z^{-1}} = 1 - \frac{0.25z^{-1}}{1-0.75z^{-1}}$$

$$h[n] = \delta[n] - 0.25 \cdot (0.75)^{n-1} u[n-1]$$

Impulse L : FIR with three non-zero values at $n=0, 1, 2$, two positive & one negative

$$S_7, \quad y[n] = x[n] + \frac{1}{4} x[n-1] - \frac{3}{4} x[n-2]$$

$$h[n] = \delta[n] + \frac{1}{4} \delta[n-1] - \frac{3}{4} \delta[n-2]$$

Impulse M : IIR with a NEGATIVE decaying factor (since $h[n]$ oscillates).

$$S_3, \quad y[n] = -0.75 y[n-1] + x[n] - x[n-1]$$

$$H(z) = \frac{1-z^{-1}}{1+0.75z^{-1}} = 1 - \frac{1.75z^{-1}}{1+0.75z^{-1}}$$

$$h[n] = \delta[n] - 1.75 \cdot (-0.75)^{n-1} u[n-1]$$

Impulse N : FIR with four non-zero values at $n=0, 1, 2, 3$

$$S_6, \quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$$

$$h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$$

Problem 12.7 (cond.)

Impulse 0 : IIR with a positive decaying factor

Since $n = 1$

$$S_2 : H(z) = \frac{1+z^{-1}}{1-0.75z^{-1}} = 1 + \frac{1.75z^{-1}}{1-0.75z^{-1}}$$

$$h[n] = \delta[n] + 1.75(0.75)^{n-1}u[n-1]$$

Impulse response of S_1 :

$$H(z) = \frac{1+z^{-1}}{1-0.4z^{-1}} = 1 + \frac{1.4z^{-1}}{1-0.4z^{-1}}$$

$$h[n] = \delta[n] + 1.4 \cdot (0.4)^{n-1}u[n-1]$$

Impulse response of S_3 :

$$H(z) = 1 - z^{-1} + z^{-2}$$

$$h[n] = \delta[n] - \delta[n-1] + \delta[n-2]$$

Problem 12.8

Response A : $H(e^{j\omega}) = 0 \& |H(e^{j\pm\pi})| = 1.1^+$

$$S_4 : H(e^{j\omega}) = 0, H(e^{j\pm\pi}) = 8/7$$

Response B : $H(e^{j\pm\pi}) = 0 \& |H(e^{j\omega})| = 8$ (Maximum)

$$S_2 : H(e^{j\pm\pi}) = 0, H(e^{j\omega}) = 8$$

Response C : $H(e^{j0.3\pi}) = 0, H(e^{j\pm\pi}) = 3$ (Maximum)

$$S_5 : H(z) = 1 - z^{-1} + z^{-2}, H(e^{j\omega}) = 1 - e^{-j\omega} + e^{-j2\omega}$$
$$H(e^{j\pm\pi}) = 3, H(e^{j\pm\pi/3}) = 0$$

Response D : $H(e^{j\pm\pi}) = 0, H(e^{j\omega}) = 3.5$ (Maximum)

$$S_1 : H(z) = \frac{1 + z^{-1}}{1 - 0.4z^{-1}}, H(e^{j\omega}) = \frac{1 + e^{-j\omega}}{1 - 0.4e^{-j\omega}}$$

$$H(e^{j\omega}) = 20/6 (\approx 3.3), H(e^{j\pm\pi}) = 0$$

Response E : $H(e^{j\omega}) = 0$ at $\omega = \pm 0.5\pi, 0$

$$H(e^{j\pi}) = 4 \text{ (max)}$$

$$S_6 : H(e^{j\omega}) = 1 - e^{-j\omega} + e^{-j2\omega} - e^{-j3\omega}$$
$$= (1 - e^{-j\omega})(1 - e^{j\pi/2} e^{-j\omega})(1 - e^{-j3\pi/2} e^{-j\omega})$$

Response F : $H(e^{j\pm\pi}) = 0, H(e^{j\omega}) = 0.5$

$$S_7 : H(e^{j\omega}) = 1 + \frac{1}{4}e^{-j\omega} - \frac{3}{4}e^{-j2\omega}$$

$$= (1 + e^{j\omega})(1 - \frac{3}{4}e^{-j\omega})$$

Problem 12.8

To derive a formula for $|H(e^{j\hat{\omega}})|$ it is easiest to do $|H(e^{j\hat{\omega}})|^2$ and then take the square root.

Also, most of the time we have terms like $1+ae^{-j\hat{\omega}}$, so

$$\begin{aligned} |1+ae^{-j\hat{\omega}}|^2 &= (1+ae^{-j\hat{\omega}})(1+ae^{-j\hat{\omega}})^* \xrightarrow{\text{conjugate}} \\ &= (1+ae^{-j\hat{\omega}})(1+ae^{+j\hat{\omega}}) \\ &= 1+a^2 + ae^{-j\hat{\omega}} + ae^{+j\hat{\omega}} = 1+a^2 + 2a\cos\hat{\omega} \end{aligned}$$

$$S_1: H(z) = \frac{1+z^{-1}}{1-0.4z^{-1}} \Rightarrow H(e^{j\hat{\omega}}) = \frac{1+e^{-j\hat{\omega}}}{1-0.4e^{-j\hat{\omega}}}$$

$$\Rightarrow |H(e^{j\hat{\omega}})| = \sqrt{\frac{2+2\cos\hat{\omega}}{1.16-0.8\cos\hat{\omega}}}$$

$$S_2: H(e^{j\hat{\omega}}) = \frac{1+e^{-j\hat{\omega}}}{1-0.75e^{-j\hat{\omega}}} \Rightarrow |H(e^{j\hat{\omega}})| = \sqrt{\frac{2+2\cos\hat{\omega}}{1.5625-1.5\cos\hat{\omega}}}$$

$$S_3: H(e^{j\hat{\omega}}) = \frac{1-e^{-j\hat{\omega}}}{1+0.75e^{-j\hat{\omega}}} \Rightarrow |H(e^{j\hat{\omega}})| = \sqrt{\frac{2-2\cos\hat{\omega}}{1.5625+1.5\cos\hat{\omega}}}$$

$$S_4: H(e^{j\hat{\omega}}) = \frac{1-e^{-j\hat{\omega}}}{1-0.75e^{-j\hat{\omega}}} \Rightarrow |H(e^{j\hat{\omega}})| = \sqrt{\frac{2-2\cos\hat{\omega}}{1.5625-1.5\cos\hat{\omega}}}$$

$$S_5: H(e^{j\hat{\omega}}) = 1-e^{-j\hat{\omega}}+e^{-j2\hat{\omega}} = e^{-j\hat{\omega}}(-1+2\cos\hat{\omega}) \Rightarrow |H(e^{j\hat{\omega}})| = |2\cos\hat{\omega}-1|$$

$$S_6: H(e^{j\hat{\omega}}) = 1-e^{-j\hat{\omega}}+e^{-j2\hat{\omega}}-e^{-j3\hat{\omega}} = e^{-j1.5\hat{\omega}}(2j\sin(1.5\hat{\omega}) - 2j\sin(0.5\hat{\omega}))$$

$$|H(e^{j\hat{\omega}})| = 2|\sin(1.5\hat{\omega}) - \sin(0.5\hat{\omega})|$$

$$S_7: H(e^{j\hat{\omega}}) = 1 + \frac{1}{4}e^{-j\hat{\omega}} - \frac{3}{4}e^{-j2\hat{\omega}}$$

$$|H(e^{j\hat{\omega}})|^2 = -\frac{3}{4}e^{-j2\hat{\omega}} + \frac{1}{16}e^{-j\hat{\omega}} + \frac{13}{8} + \frac{1}{16}e^{+j\hat{\omega}} - \frac{3}{4}e^{+j2\hat{\omega}}$$

$$\Rightarrow |H(e^{j\hat{\omega}})| = \sqrt{13/8 + \frac{1}{8}\cos\hat{\omega} - \frac{3}{2}\cos 2\hat{\omega}}$$

$$S_8: H(e^{j\hat{\omega}}) = \frac{1}{3}(1-e^{-j\hat{\omega}})^3 \Rightarrow |H(e^{j\hat{\omega}})| = \frac{8}{3}|\sin(\frac{\hat{\omega}}{2})|^{3/2}$$

$$1-e^{-j\hat{\omega}} = e^{-j\hat{\omega}/2}(2j\sin\frac{\hat{\omega}}{2})$$