A Review of Matrix Multiplication

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Recall that before a matrix A can multiply another matrix B, the number of columns in A must equal the number of rows in B. For example, if A is $m \times n$, then B must be $n \times l$. The product of these two matrices would be $m \times l$. Obviously, matrix multiplication is *not* commutative, as the product $B \cdot A$ is undefined for $l \neq m$.

To generate the first element in the product of two matrices, A and B, simply take the first row of A and multiply *point by point* with the first column of B, then sum. For example, if

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{bmatrix}$$

then the first element of $C = A \cdot B$ would be

$$c_{1,1} = a_{1,1}b_{1,1} + a_{1,2}b_{2,1} + a_{1,3}b_{3,1}.$$

$c_{1,2}$ is found by taking the first row of A and multiplying point by point with the *second* column of B then summing. $c_{2,1}$ is found by taking the *second* row of A and multiplying point by point with the *first* column of B then summing. Finally, $c_{2,2}$ is found by multiplying the second row of A point by point with the second column of B then summing. The result would be:

$$C = \begin{bmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{bmatrix} = \begin{bmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} + a_{1,3}b_{3,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} + a_{1,3}b_{3,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} + a_{2,3}b_{3,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} + a_{2,3}b_{3,2} \end{bmatrix}$$

Two useful results are the *outer product:*

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} a_1 & a_1 & a_1 & a_1 \\ a_2 & a_2 & a_2 & a_2 \\ a_3 & a_3 & a_3 & a_3 \end{bmatrix}$$

and the *inner product:*

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = a_1 + a_2 + a_3 + a_4$$

There may be times when you want to multiply matrices element by element, that is $c_{i,j} = a_{i,j} \cdot b_{i,j}$. Obviously, the matrices would have to be the same size to do this. In MATLAB this is accomplished using the .* operation. For example if A and B are both 2×2:

$$C = P. * Q = Q. * P = \begin{bmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{bmatrix} = \begin{bmatrix} p_{1,1} \cdot q_{1,1} & p_{1,2} \cdot q_{1,2} \\ p_{2,1} \cdot q_{2,1} & p_{2,2} \cdot q_{2,2} \end{bmatrix}$$

Also, remember in MATLAB that `cos()` and `exp()` are applied to each element of a matrix. For example,

$$\cos(A) = \begin{bmatrix} \cos(a_{1,1}) & \cos(a_{1,2}) & \cdots & \cos(a_{1,n}) \\ \cos(a_{2,1}) & \cos(a_{2,2}) & \cdots & \cos(a_{2,n}) \\ \vdots & \vdots & \vdots & \vdots \\ \cos(a_{m,1}) & \cos(a_{m,2}) & \cdots & \cos(a_{m,n}) \end{bmatrix}$$

We can exploit the inner product idea to sum up the cosines in the matrix:

$$\begin{bmatrix} \cos(a_{1,1}) & \cos(a_{1,2}) & \cdots & \cos(a_{1,n}) \\ \cos(a_{2,1}) & \cos(a_{2,2}) & \cdots & \cos(a_{2,n}) \\ \vdots & \vdots & \vdots & \vdots \\ \cos(a_{m,1}) & \cos(a_{m,2}) & \cdots & \cos(a_{m,n}) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix} = \sum_{k=1}^{n} A_k \cos(a_{m,k})$$