HOMEWORK #7

SOLUTIONS

ECE 2025
SPRING'01

7.1) See solution to prob. 6.1, Fall 2000.

7.2) See solution to prob. 6.1, Spring 1999.

7.3) a.i) The system is linear because:

$$x_1[n] \to y_1[n] = 2x_1[n+2] + 6x_1[n] + 2x_1[n-2]$$

$$x_2[n] \to y_2[n] = 2x_2[n+2] + 6x_2[n] + 2x_2[n-2]$$

$$c_1x_1[n] + c_2x_2[n] \longrightarrow y_3[n] = 2\left(c_1x_1[n+2] + \right.$$

$$\left. + c_2x_2[n+2]\right) + 6\left(c_1x_1[n] + c_2x_2[n]\right) +$$

$$+ 2\left(c_1x_1[n-2] + c_2x_2[n-2]\right) =$$

$$= c_1\left(2x_1[n+2] + 6x_1[n] + 2x_1[n-2]\right) +$$

$$+ c_2\left(2x_2[n+2] + 6x_2[n] + 2x_1[n-2]\right) =$$

$$= c_1y_1[n] + c_2y_2[n].$$

ii) The system is time-invariant because:

$$x[n-n_0] \to 2x[n+2-n_0] + 6x[n-n_0] +$$

$$+ 2x[n-2-n_0] = y[n-n_0].$$

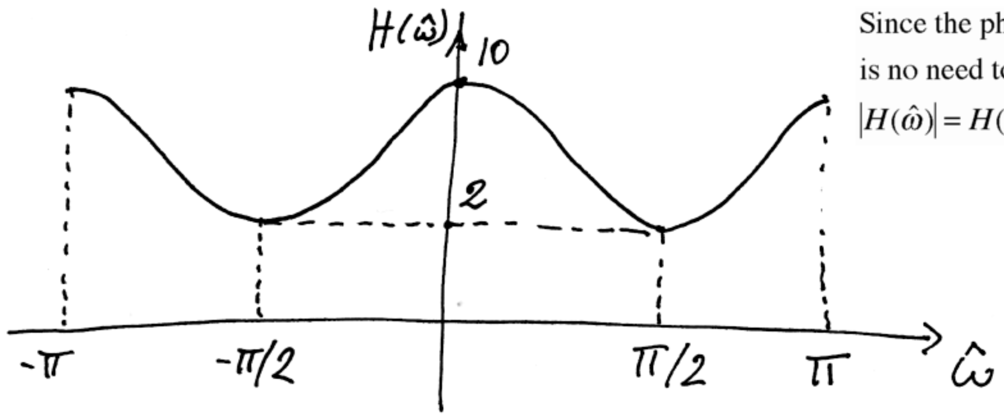
iii) The system is not causal because $y[n]$ depends on $x[n+2]$.

b) $h[n] = 2\delta[n+2] + 6\delta[n] + 2\delta[n-2]$

$$\Rightarrow H(\hat{\omega}) = 2e^{j2\hat{\omega}} + 6 + 2e^{-j2\hat{\omega}} =$$

$$= 6 + 4\cos(2\hat{\omega})$$

c)



Since the phase is zero, there is no need to plot the phase: $|H(\hat{\omega})| = H(\hat{\omega})$

d) $H(0) = 10 \qquad H(0.5\pi) = 2 = H(-0.5\pi)$

$$10 \longrightarrow H(0)\cdot 10 = 100$$

$$10\cos\left[\frac{\pi}{2}(n-1)\right] = 5e^{-j\frac{\pi}{2}}e^{j\frac{\pi}{2}n} + 5e^{j\frac{\pi}{2}}e^{-j\frac{\pi}{2}n}$$

$$\longrightarrow 5e^{-j\frac{\pi}{2}}H(0.5\pi)e^{j\frac{\pi}{2}n} + 5e^{j\frac{\pi}{2}}H(-0.5\pi)e^{-j\frac{\pi}{2}n} =$$

$$= 10e^{-j\frac{\pi}{2}}e^{j\frac{\pi}{2}n} + 10e^{j\frac{\pi}{2}}e^{-j\frac{\pi}{2}n} =$$

$$= 20\cos\left[\frac{\pi}{2}(n-1)\right]$$

$$y_1[n] = 100 - 20\cos\left[\frac{\pi}{2}(n-1)\right]$$

7.4) a.i) The system is <u>not</u> linear because:

$$x[n] \rightarrow y[n] = (x[n+1])^3$$

$$c\,x[n] \rightarrow y_1[n] = (c\,x[n+1])^3 =$$

$$= c^3 (x[n+1])^3 = c^3 y[n] \neq c\,y[n].$$

ii) The system is time-invariant because:

$$x[n-n_0] \rightarrow y_1[n] = (x[n-n_0+1])^3 =$$

$$= y[n-n_0].$$

iii) The system is <u>not</u> causal because $y[n]$ depends on $x[n+1]$.

b) $$y_1[n] = \left(e^{j0.6\pi n} + e^{-j0.6\pi n}\right)^3 =$$

$$= e^{j1.8\pi n} + 3e^{j1.2\pi n} e^{-j0.6\pi n} +$$

$$+ 3e^{j0.6\pi n} e^{-j1.2\pi n} + e^{-j1.8\pi n}$$

Note that $1.8\pi > \pi$, so:

$$e^{j1.8\pi n} = e^{j(1.8\pi - 2\pi)n} = e^{-j0.2\pi n}$$

Similarly: $e^{-j1.8\pi n} = e^{j0.2\pi n}$

So: $y_1[n] = 2\cos(0.2\pi n) + 6\cos(0.6\pi n)$

7.5) a) $h_1[n] = \delta[n] - \alpha\,\delta[n-1]$

(by definition, the impulse response is the system's output when the input is $\delta[n]$).

In the difference equation, set $x[n] = \delta[n]$

(b) Use numerical convolution

$$\begin{array}{cccccc}
1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \\
1 & -\alpha & & & & \\
\hline
1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \\
 & -\alpha & -\alpha^2 & -\alpha^3 & -\alpha^4 & -\alpha^5 \\
\hline
1 & 0 & 0 & 0 & 0 & -\alpha^5
\end{array}$$

n=5 column

Thus, $h[n] = \delta[n] - \alpha^5\delta[n-5]$

(c) In part (b) $L=5$, so we suspect that

$h[n] = \delta[n] - \alpha^L\delta[n-L]$ for any L

7.5 (c) In part (b) $L=5$, so we suspect that

$$h[n] = \delta[n] - \alpha^L \delta[n-L]$$

for any L

Here is a general derivation:

$$h_1[n] * h_2[n] = (\delta[n] - \alpha\delta[n-1]) * (\alpha^n u[n] - \alpha^n u[n-L])$$

$$= \delta[n] * \alpha^n u[n] - \alpha\delta[n-1] * \alpha^n u[n] - \delta[n] * \alpha^n u[n-L] + \alpha\delta[n-1] * \alpha^n u[n-L]$$

$$= \alpha^n u[n] - \alpha\alpha^{n-1} u[n-1] - \alpha^n u[n-L] + \alpha\alpha^{n-1} u[n-1-L]$$

NOTE: $\delta[n-1] * x[n] = x[n-1]$

$$= \alpha^n \big(\underbrace{u[n] - u[n-1]}_{=\delta[n]}\big) - \alpha^n \big(\underbrace{u[n-L] - u[n-1-L]}_{\delta[n-L]}\big)$$

$$= \alpha^n \delta[n] - \alpha^n \delta[n-L]$$

Non-Zero only when $n=L$

$$= \delta[n] - \alpha^L \delta[n-L]$$

d) The impulse response of the overall system is:

$$h[n] = \delta[n] - \alpha^L \delta[n-L]$$

so: $y[n] = x[n] - \alpha^L x[n-L]$

e) There is no finite value of L such that $y[n] = x[n]$ for $\alpha > 0$. However, if $\alpha < 1$ then $\alpha^L \to 0$ as $L \to +\infty$, so $y[n] = x[n]$ if $L = +\infty$, i.e. $h_2[n] = \alpha^n u[n]$.

7.6) a) $H_1(\hat{\omega}) = 1 - e^{-j\hat{\omega}}$

b) $H_2(\hat{\omega}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$

(c) From 7.5(d) with $\alpha = 1$ and $L = 5$

$$h[n] = \delta[n] - \delta[n-5]$$

Therefore, the filter coefficients are: $\{b_k\} = \{1, 0, 0, 0, 0, -1\}$

$$\Rightarrow \mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = 1 - e^{-j\hat{\omega}5}$$

d) $$H_1(\hat{\omega}) H_2(\hat{\omega}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

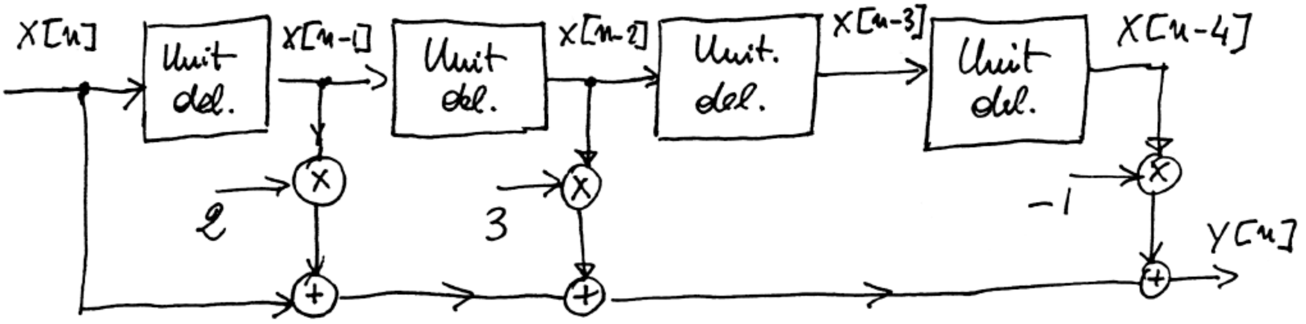
$$- e^{-j\hat{\omega}}\left(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}\right)$$

$$= 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$- e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} - e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$= 1 - e^{-j5\hat{\omega}} = H(\hat{\omega}).$$

7.7) a)

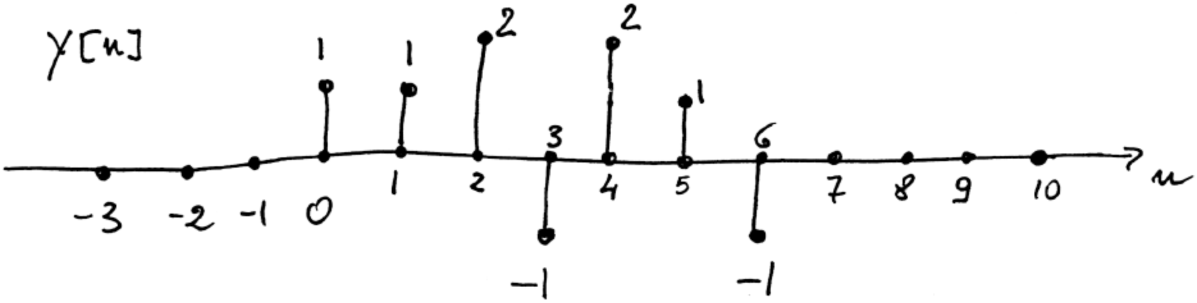


b) $h[n] = \delta[n] + 2\,\delta[n-1] + 3\,\delta[n-2] - \delta[n-4]$

(cf. 7.5.a).

c)

$$\begin{array}{ccccccc}
1 & 2 & 3 & 0 & -1 & & \\
1 & -1 & 1 & & & & \\
\hline
1 & 2 & 3 & 0 & -1 & & \\
 & -1 & -2 & -3 & 0 & 1 & \\
 & & 1 & 2 & 3 & 0 & -1 \\
\hline
1 & 1 & 2 & -1 & 2 & 1 & -1
\end{array}$$



d)

$$
\begin{array}{ccccccc}
1 & -1 & 1 & & & & \\
1 & 2 & 3 & 0 & -1 & & \\
\hline
1 & -1 & 1 & & & & \\
 & 2 & -2 & 2 & & & \\
 & & 3 & -3 & 3 & & \\
 & & & 0 & 0 & 0 & \\
 & & & & -1 & 1 & -1 \\
\hline
1 & 1 & 2 & -1 & 2 & 1 & -1
\end{array}
$$

Same as in (c).

7.8) a) $H(\hat{\omega}) = (1+0.8e^{-j\hat{\omega}})(1 - \cancel{e^{j\frac{\pi}{2}}}e^{-j\hat{\omega}} +$

$- \cancel{e^{-j\frac{\pi}{2}}}e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) =$

$= 1 + 0.8e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + 0.8e^{-j3\hat{\omega}}$

$y[n] = x[n] + 0.8\,x[n-1] + x[n-2] +$

$+ 0.8\,x[n-3]$

b) $h[n] = \delta[n] + 0.8\,\delta[n-1] + \delta[n-2] +$

$+ 0.8\,\delta[n-3].$

c) If $x[n] = X_0 e^{j\hat{\omega}n}$, then $y[n] = X_0 H(\hat{\omega}) e^{j\hat{\omega}n}$, so $y[n] \equiv 0$ if $H(\hat{\omega}) = 0$. From the factored form of $H(\hat{\omega})$ given in eqn. (3), $H(\hat{\omega}) = 0$ when:

$$1 - e^{-j\frac{\pi}{2}} e^{-j\hat{\omega}} = 1 - e^{-j(\hat{\omega} + \frac{\pi}{2})} = 0 \Rightarrow \hat{\omega} = -\frac{\pi}{2}$$

$$1 - e^{j\frac{\pi}{2}} e^{-j\hat{\omega}} = 1 - e^{-j(\hat{\omega} - \frac{\pi}{2})} = 0 \Rightarrow \hat{\omega} = \frac{\pi}{2}$$

($1 + 0.8 e^{-j\hat{\omega}} \neq 0$ for all values of $\hat{\omega}$ because $|0.8 e^{-j\hat{\omega}}| = 0.8 < 1$.)

So $H(\hat{\omega}) = 0$ for $\hat{\omega} = \pm \frac{\pi}{2}$.

d) $3 \rightarrow H(0) \cdot 3 = 3.6 \cdot 3 = 10.8$

$$\delta[n-3] \rightarrow h[n-3] = \delta[n-3] + 0.8\,\delta[n-4] + \delta[n-5] + 0.8\,\delta[n-6]$$

$$e^{j0.5\pi n} \rightarrow H(\tfrac{\pi}{2}) e^{j0.5\pi n} = 0.$$

So $y[n] = 10.8 + \delta[n-3] + 0.8\,\delta[n-4] + \delta[n-5] + 0.8\,\delta[n-6]$.