Problem Set #5
ECE 2025 Spring 2001

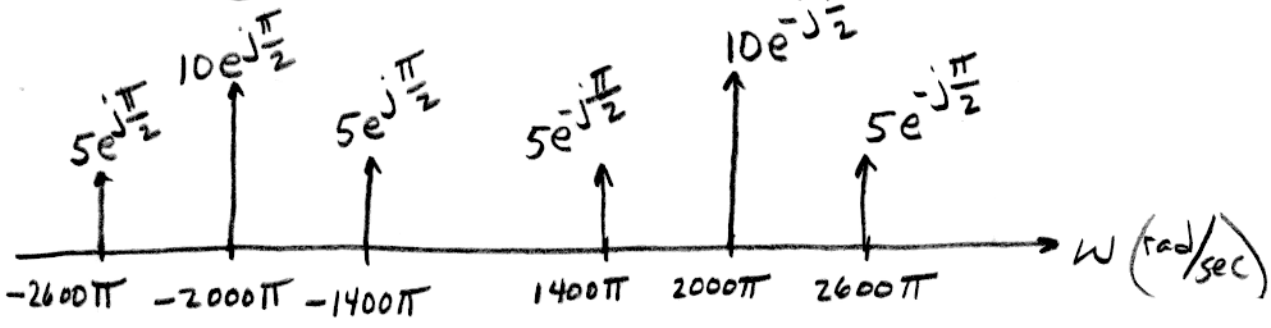
Problem 5.1

a) $x(t) = 20\cos(2000\pi t - \pi/2) + 20\cos(2000\pi t - \pi/2)\cos(600\pi t)$

$$x(t) = 20\cos\left(2000\pi t - \frac{\pi}{2}\right) + 10\cos\left(1400\pi t - \frac{\pi}{2}\right) + 10\cos\left(2600\pi t - \frac{\pi}{2}\right)$$

Using $\cos(\omega_1 t+\phi_1)\cos(\omega_2 t+\phi_2) = \frac{1}{2}\cos\left((\omega_1+\omega_2)t+\phi_1+\phi_2\right) + \frac{1}{2}\cos\left((\omega_1-\omega_2)t+(\phi_1-\phi_2)\right)$

$$x(t) = 10e^{-j\frac{\pi}{2}}e^{j2000\pi t} + 10e^{j\frac{\pi}{2}}e^{-j2000\pi t} + 5e^{-j\pi/2}e^{j1400\pi t} + 5e^{j\pi/2}e^{-j1400\pi t} + 5e^{-j\pi/2}e^{j2600\pi t} + 5e^{j\pi/2}e^{-j2600\pi t}$$



$\gcd(1400\pi, 2000\pi, 2600\pi) = 200\pi$ rad/s

$\Rightarrow$ Fundamental Period $= 2\pi/200\pi = \frac{1}{100}$ secs.

$x(t)$ is <u>PERIODIC</u>

b) Minimum Sampling Rate, f_s, must be twice the highest frequency found within the $x(t)$ signal.
Max. frequency of $x(t)$ is 2600π

$$\omega_{MAX} = 2600\pi \Rightarrow f_{MAX} = 1300 \text{ Hz} \Rightarrow f_s = 2f_{MAX} = 2600 \text{ Hz}$$

Problem 5.1 (cont.)

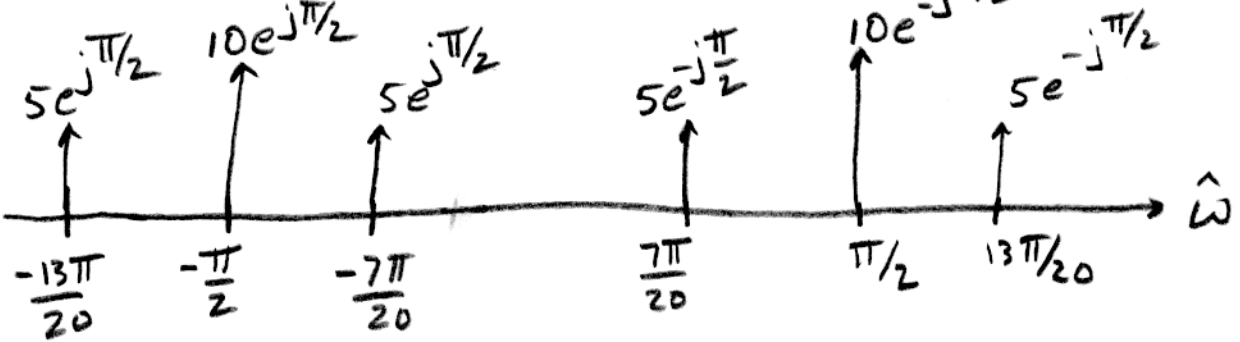
c) Spectrum for $x[n]$ when $f_s = 4000$ Hz

$$\hat{\omega} = 2\pi \frac{f}{f_s} = \frac{2\pi f}{4000} = \frac{\omega}{4000}$$

for $\omega = 1400\pi \Rightarrow \hat{\omega} = 7\pi/20$

for $\omega = 2000\pi \Rightarrow \hat{\omega} = \pi/2$

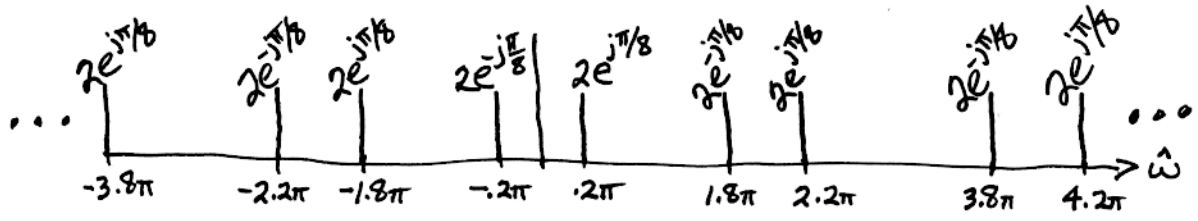
for $\omega = 2600\pi \Rightarrow \hat{\omega} = 13\pi/20$



Prob 5.2

(a) $x[n] = 4\cos(0.2\pi n + \pi/8) \quad \Rightarrow \quad \hat{\omega} = 0.2\pi$

Draw the discrete-time spectrum with all of its aliases. In other words, add $2\pi\ell$ to $\hat{\omega}$.



Each $\hat{\omega}$ can be mapped back to a cont-time freq, ω.

The components at $\hat{\omega} = \pm 0.2\pi$:

$$\omega = \hat{\omega} f_s = (0.2\pi)(8000) = 1600\pi \text{ rad/sec}$$
or, 800 Hz.

$$x_1[n] = 4\cos(0.2\pi n + \pi/8)$$
$$\rightarrow x_1(t) = 4\cos(1600\pi t + \pi/8)$$

The components at $\hat{\omega} = \pm 1.8\pi$

$$\omega = \hat{\omega} f_s = (1.8\pi)(8000) = 14{,}400\pi \text{ rad/sec}$$
or, 7200 Hz

$$x_2[n] = 4\cos(0.2\pi n + \pi/8)$$
$$= 4\cos(1.8\pi n - \pi/8)$$

These are the same signal

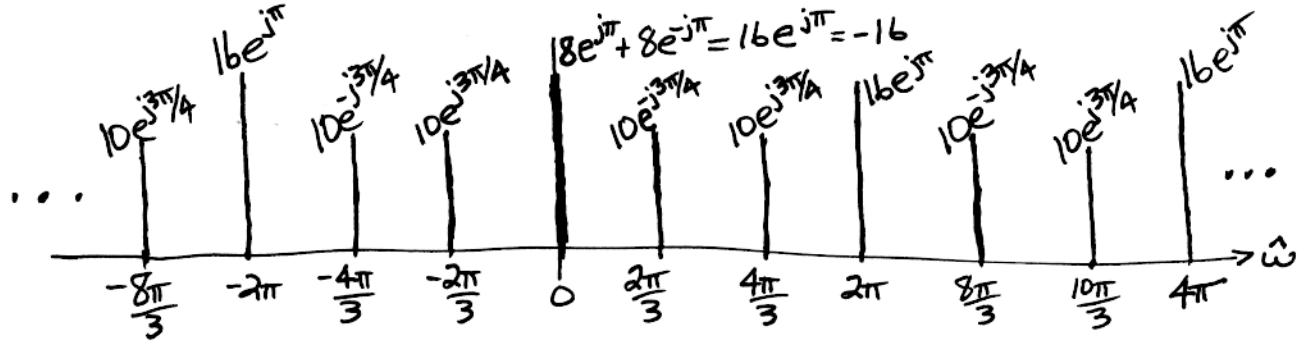
$$\rightarrow x_2(t) = 4\cos(14{,}400\pi t - \pi/8)$$

(b) $f_{MAX} = 150$ Hz $\Rightarrow$ $f_s \geq 2 f_{MAX} = 300$ samples/sec

(c) $\hat{\omega} = \omega / f_s \Rightarrow$ $f = 50$ Hz becomes $\hat{\omega} = 2\pi(50)/150 = 2\pi/3$

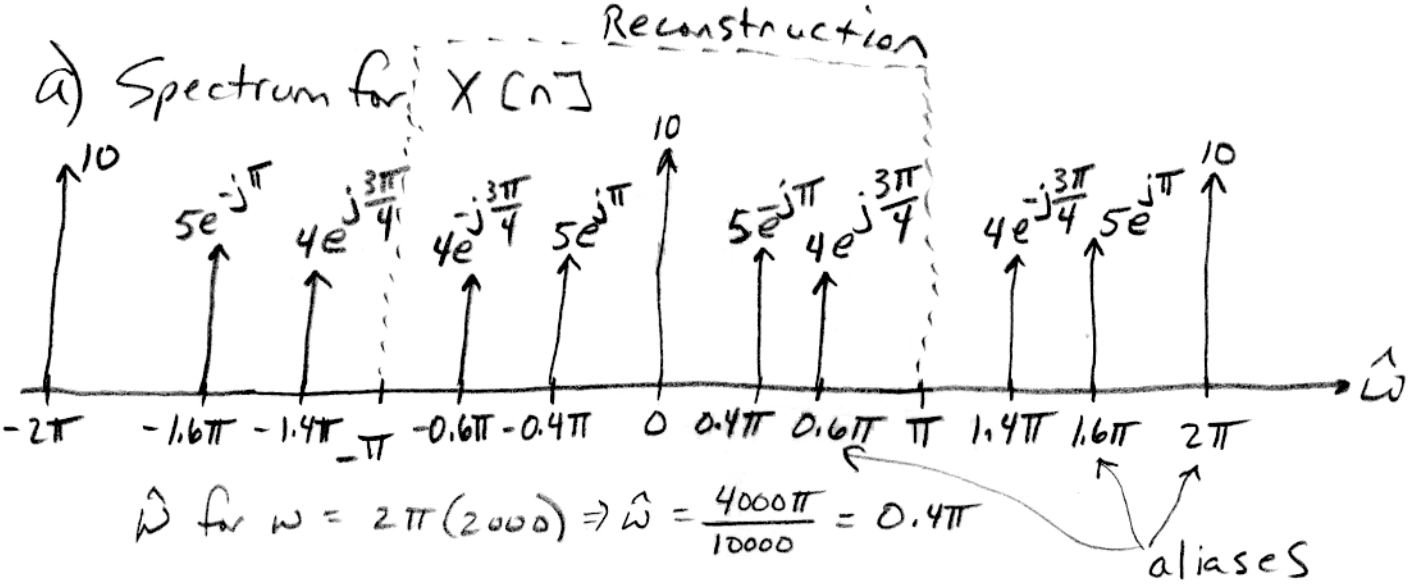
$f = 150$ Hz becomes $\hat{\omega} = 2\pi(150)/150 = 2\pi$

BUT $\hat{\omega} = 2\pi$ is the same as $\hat{\omega} = 0$



Problem 5.3

a) Spectrum for $x[n]$



$\hat{\omega}$ for $\omega = 2\pi(2000) \Rightarrow \hat{\omega} = \frac{4000\pi}{10000} = 0.4\pi$

b) If $f_{s_{D-C}} = 10{,}000$ Hz.

Reconstruct signal from freq. components between $-\pi \rightarrow \pi$ on the $\hat{\omega}$ scale. Therefore, frequency components are:

$\hat{\omega} = 0.4\pi \Rightarrow f = \frac{0.4\pi}{2\pi} f_s = 2{,}000$ Hz

$\hat{\omega} = 0.6\pi \Rightarrow f = 3{,}000$ Hz

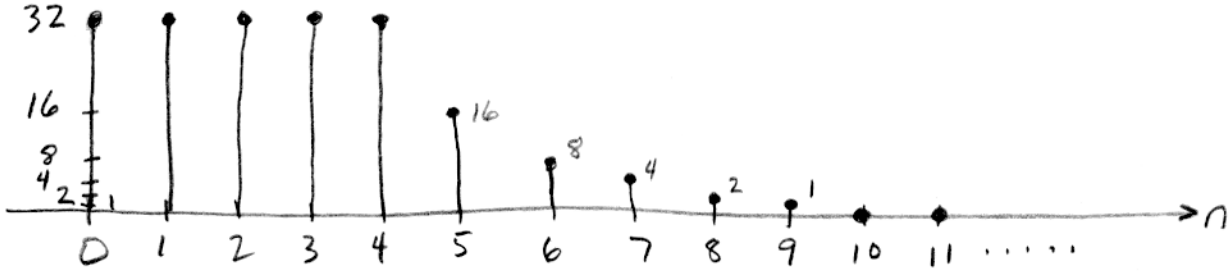
c) If $f_{s_{D-C}} = 20{,}000$ Hz, then

For $\hat{\omega} = 0.4\pi \Rightarrow f = 4{,}000$ Hz

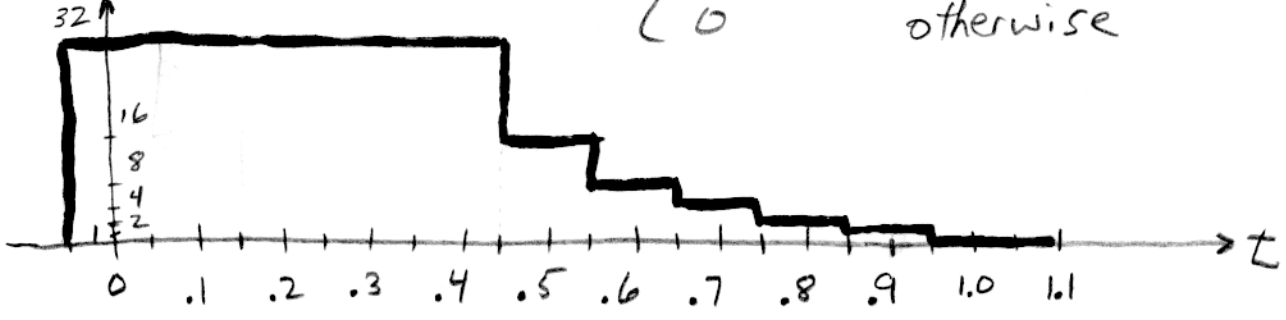
For $\hat{\omega} = 0.6\pi \Rightarrow f = 6{,}000$ Hz

Problem 5.4

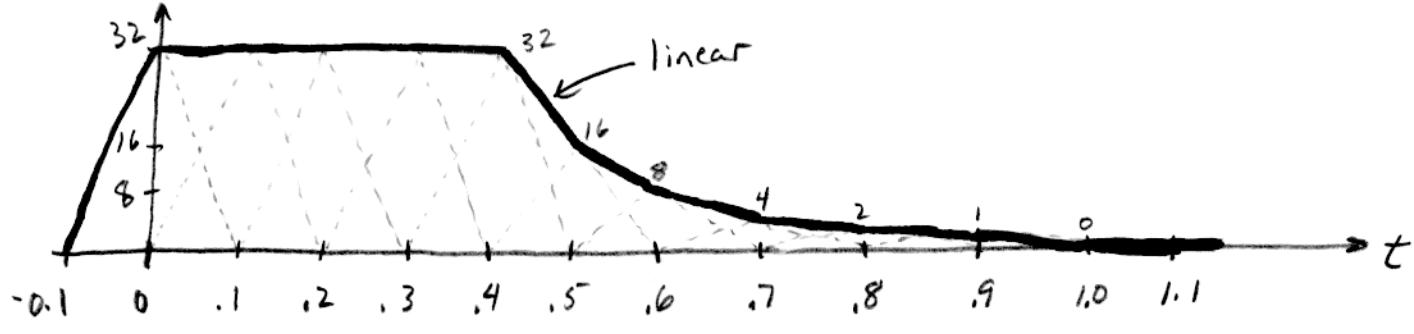
a) Plot $y[n]$ vs n



b) Plot $y(t)$ for $p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$



c) Plot $y(t)$ for $p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$



Problem 5.5

(a) 15 revolutions/sec $= (2\pi \text{ rad/rev})(15 \text{ rev/s}) = 30\pi$ rad/sec

Clockwise $\Rightarrow \omega = -30\pi$ rad/s

Rotating phasor position $= re^{j\varphi}e^{-j30\pi t}$

r = length

φ = initial phase

(b) n flashes per second

stand still $\Rightarrow \hat{\omega} = 0$ or $2\pi\ell$ ← ℓ = integer

$\hat{\omega} = \omega/f_s \Rightarrow 2\pi\ell = \frac{-30\pi}{n}$

$\therefore n = \frac{-30\pi}{2\pi\ell} = \frac{-15}{\ell}$ ← Choose ℓ so that n = integer

$\ell = -1, -3, -5, -15 \Rightarrow n = 15, 5, 3, 1$

possible flash rates 15 flash/s, 5 fl/s, 3 fl/s, 1 fl/sec

(c) Flashing period = 100 msec $\Rightarrow f_s = 1/0.1 = 10$ flash/s

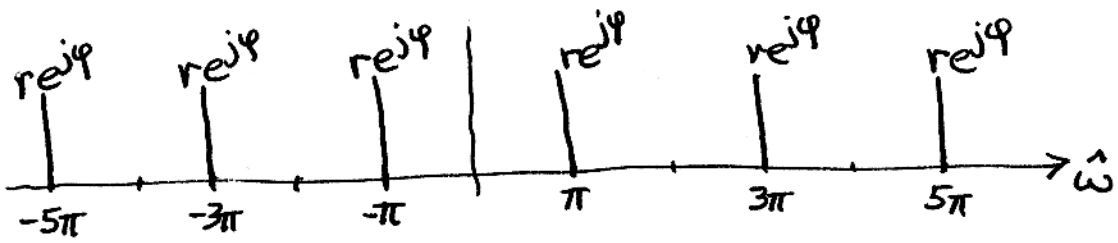
Use rotating phasor from part (a) and replace "t" with "n/f_s".

$$re^{j\varphi}e^{-j30\pi\, n/10} = re^{j\varphi}e^{-j3\pi n} = re^{j\varphi}e^{-j3\pi n}e^{j2\pi\ell n}$$

$$= re^{j\varphi}e^{j\pi(2\ell-3)n}$$

If $\ell = 2$, the rotating part of the answer is $e^{j\pi n}$, so the dot move by π rads each flash. There are 10 flashes/sec, so the dot will make 5 revs/sec.

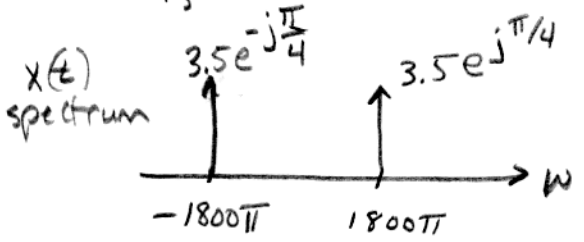
(d)

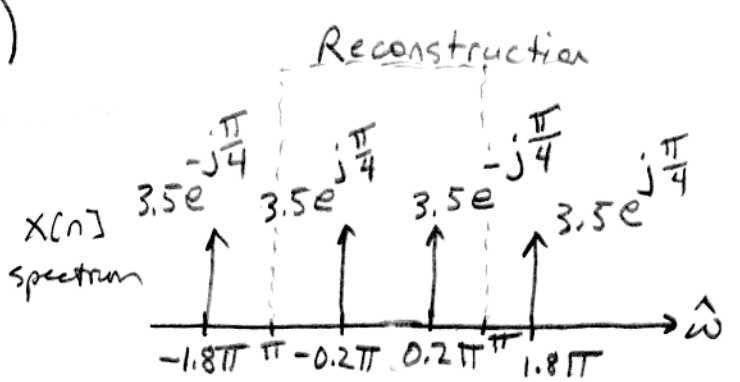


Problem 5.6

a) $x(t) = 7\cos(1800\pi t + \frac{\pi}{4})$

$f_s = 1000$ Hz





Reconstruct using signals in the range $-\pi \to \pi$ (this is the spectrum for $y[n]$

$y[n] = 7\cos(0.2\pi n - \frac{\pi}{4})$

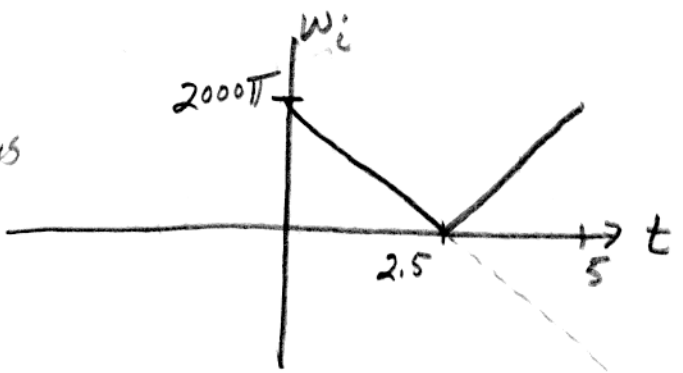
$\Rightarrow y(t) = 7\cos(200\pi t - \frac{\pi}{4})$

b) $x(t) = \cos(2000\pi t - 400\pi t^2)$ for $0 \le t \le 5$ sec

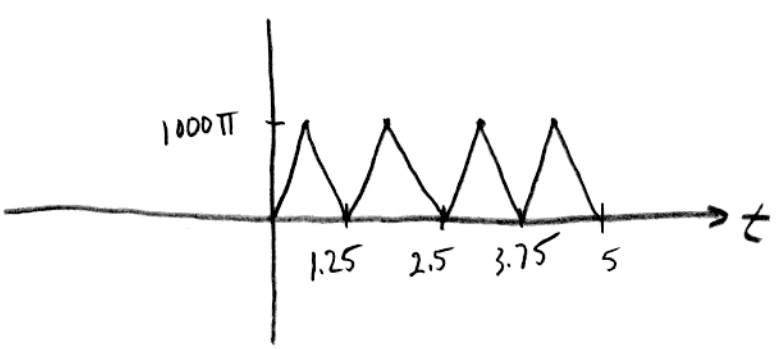
$f_s = 1000$ Hz $\quad w_i(t) = 2000\pi - 800\pi t$ $\quad$ At $t = 0 \Rightarrow w_i = 2000\pi$

$t = T_s \Rightarrow w_i =$

x(t) instantaneous freq.



y(t) instantaneous freq.



Note: Same as Problem 5.2, Fall 2000