ECE 2025
Spring 2001
HW# 3
Solutions

Problem 3.1

A) Frequency of DC Component ? $\emptyset$ Hz

Period of Cosine = 1.0 msec

$$\text{Frequency} = \frac{1}{\text{Period}} = \frac{1}{1\times10^{-3}} = 1000\ \text{Hz}$$

B)

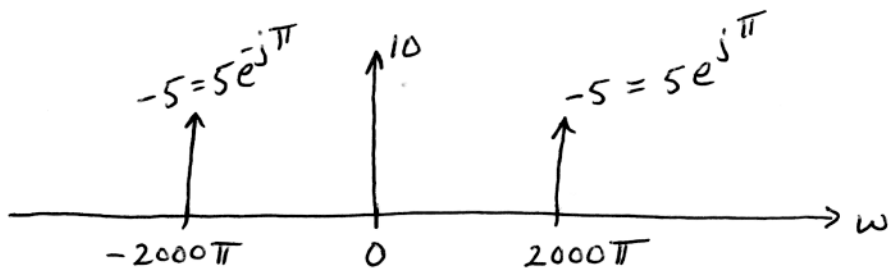
$$x(t) = 10 + 10\cos\left(2\pi(1000)t + \pi\right)$$

C)

$$x(t) = 10 + 5e^{j2000\pi t}e^{j\pi} + 5e^{-j2000\pi t}e^{-j\pi}$$

$$= 10 - 5\left(e^{j2000\pi t} + e^{-j2000\pi t}\right)$$

D)



Problem 3.2

A) $x(t) = 20\left[\sin(1000\pi t)\right]^2 = 20\left[\frac{1-\cos 2000\pi t}{2}\right]$

$$= 10 - 10\cos 2000\pi t$$

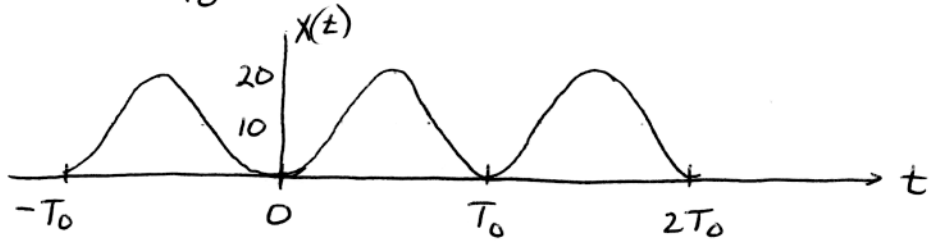
$$= 10 - 5\left(e^{j2000\pi t} + e^{-j2000\pi t}\right)$$

Problem 3.2 (continued)

B)

$$x(t) = 10 - 10\cos 2000\pi t$$

C) $\text{Period} = \frac{1}{f_0} = \frac{1}{1000} = 1 \text{ msec}$

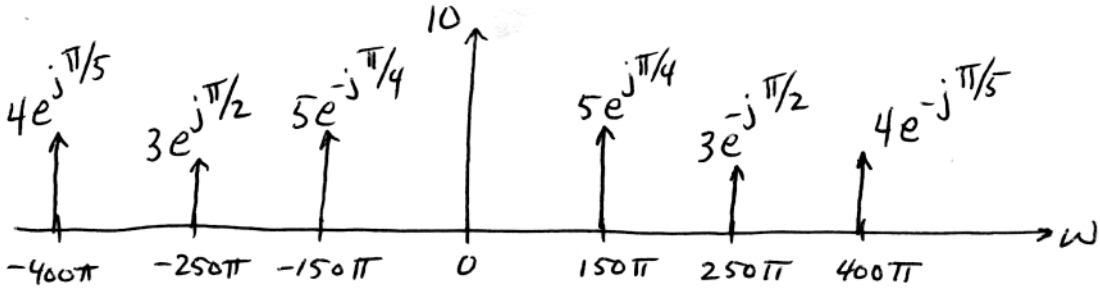


D)

Problem 3.3

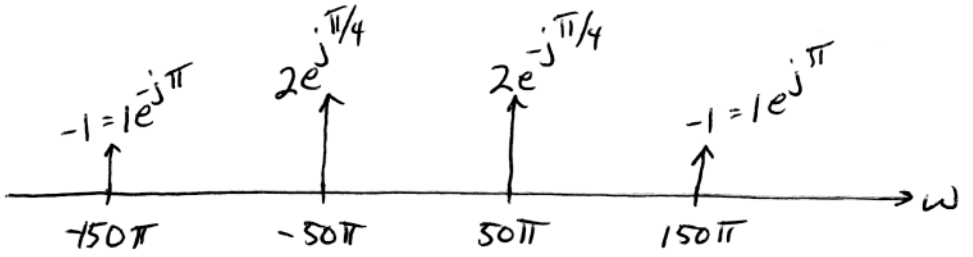
A) $x(t) = 20 + 20\cos(150\pi t + \pi/4) + 16\cos(400\pi t - \frac{\pi}{5})$

B) $y(t) = 0.5\,x(t) + 6\cos(250\pi t - \frac{\pi}{2})$



Problem 3.4

A)



B) Yes, $x(t)$ is periodic

Period is derived from the greatest common denominator of the input frequencies

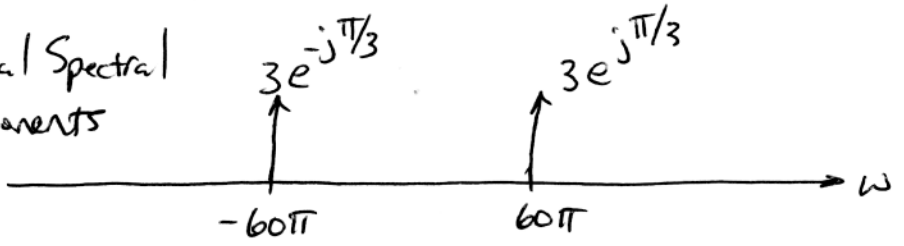
$$\omega_0 = 50\pi \Rightarrow \text{Period} = \frac{2\pi}{\omega_0} = \frac{2\pi}{50\pi} = \frac{1}{25} = 40 \text{ msec}$$

Harmonics: Fundamental + Third

C) Yes, $y(t)$ is periodic.

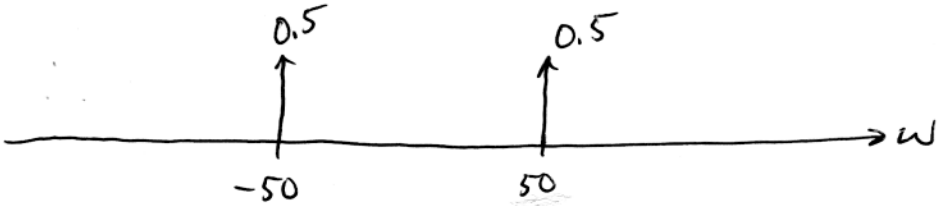
$$\text{Period} = \frac{2\pi}{\omega_0} = \frac{2\pi}{10\pi} = \frac{1}{5} = 200 \text{ msec}$$

Additional Spectral Components



D) No, $w(t)$ is not periodic. There is not a common denominator for the signal input frequencies.

Additional Spectral Components



Problem 3.5

A)

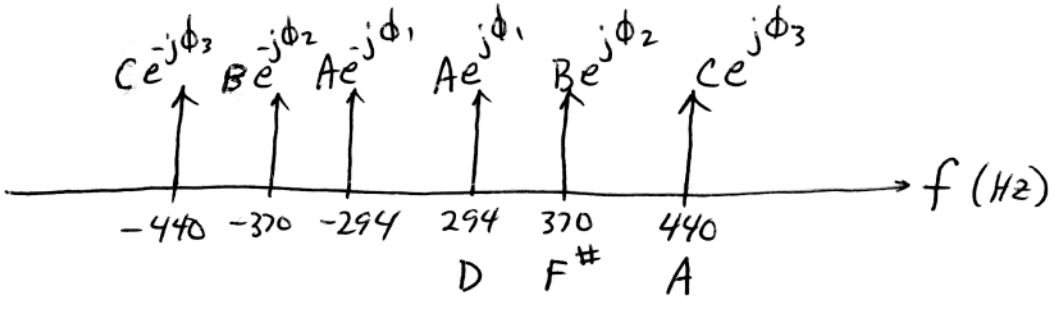
Note Name	C	C#	D	Eb	E	F	F#	G	G#	A(440)	Bb	B	C
Note Number	40	41	42	43	44	45	46	47	48	49	50	51	52
Frequency (Hz)	262	277	294	311	330	349	370	392	415	440	466	494	523

B) The ratio of frequencies for successive notes must be $2^{1/12}$ because every 12 notes corresponds to a doubling of the frequency

$$\left(2^{1/12}\right)^{12} = 2$$

Therefore, the formula is: $f = 440\, 2^{(n-49/12)}$

c)



Problem 3.6

(a) → 4
(b) → 1
(c) → 2
(d) → 5
(e) → 3

Problem 3.7

A) Instantaneous Frequency $\equiv \omega_i(t) = \frac{d}{dt}\varphi(t)$ rad/sec

$$\omega_i(t) = \frac{d}{dt}(\alpha t^2 + \beta t + \phi)$$

$$\omega_i(t) = 2\alpha t + \beta \quad \text{rad/sec}$$

$$\omega_1 = \omega_i(t=0) = 2\alpha(0) + \beta = \beta \quad \text{rad/sec}$$

$$\omega_2 = \omega_i(t=T_2) = 2\alpha T_2 + \beta$$

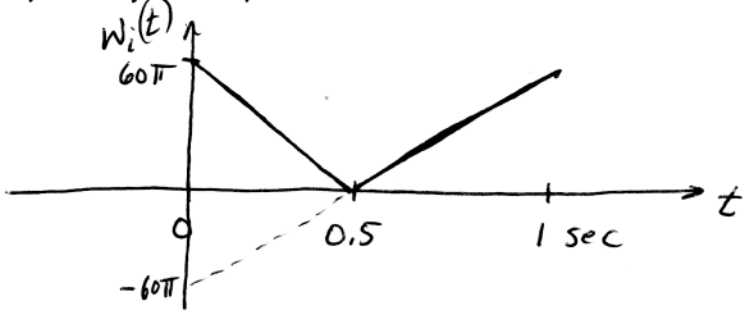
B) $x(t) = \cos[2\pi(30t^2 - 30t)]$

$$\omega_i(t) = \frac{d}{dt}[2\pi(30t^2 - 30t)]$$

$$\omega_i(t) = 120\pi t - 60\pi$$

frequency is positive for $t \geq 0.5$ sec.

C)



The calculated instantaneous frequency is negative, but the audible frequency is positive, real valued.