ECE 2025   Spring 2001
Problem Set #1

Assigned: 8-Jan-01
Due Date: Week of 15-Jan-01

Reading: In *DSP First*, Appendix A on *Complex Numbers,* pp. 378–398; and Ch. 2 on *Sinusoids,* pp. 9–43.

The web site for the course uses Web-CT:

  `webct.gatech.edu`

Your initial password for webct is the last 4 digits of your SSN (or student number), but you should change it.

$\Longrightarrow$ Please check the "Bulletin Board" daily. All official course announcements will be posted there.

ALL of the **STARRED** problems should be turned in for grading.

Some of the problems have solutions that can be found on the CD-ROM. Next week a solution to all the starred problems in this assignment will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

For the week of January 15 only, students in the **Monday recitations** should turn in HW #1 on Wednesday (17-Jan-01) at their lab time.

Several different mathematical notations can be used to represent complex numbers. In *rectangular form* we will use all of the following notations:

$$
\begin{aligned}
z &= (x, y) \\
&= x + jy \qquad \text{where } j = \sqrt{-1} \\
&= \Re e\{z\} + j\Im m\{z\}
\end{aligned}
$$

The pair (x, y) can be drawn as a vector, such that x is the horizontal coordinate and y the vertical coordinate. In *polar form* we will use these notations:

$$
\begin{aligned}
z &= |z|e^{j \arg z} \\
&= re^{j\theta} \\
&= r\angle\theta
\end{aligned}
$$

where $|z| = r = \sqrt{x^2 + y^2}$ and $\arg z = \theta = \arctan(y/x)$. Again, in a vector drawing, r is the length and θ the direction of the vector.

Euler's Formula:

$$ re^{j\theta} = r\cos\theta + j\,r\sin\theta $$

can be used to convert between Cartesian and polar forms.

Problems 1.1–1.4 should be review problems. In these problems you will manipulate some complex numbers. A calculator will be useful for this purpose, especially if it is one with complex arithmetic capability. It is convenient to learn how to use this feature. However, it is also worthwhile to be able to do the calculations by hand; i.e., it is important to *understand* what your calculator is doing!

PROBLEM 1.1:
Convert the following to polar form:

(a) $z = -j10$ (b) $z = -5$ (c) $z = (-10, 10)$

(d) $z = 2 - j2$ (e) $z = -1 + j\sqrt{3}$ (f) $z = 5$

Give numerical values for the magnitude, and the angle (or phase) in radians.

PROBLEM 1.2:
Convert the following to rectangular form:

(a) $z = 3\,e^{j(3\pi/4)}$ (c) $z = 4 \angle -(\pi/3)$

(b) $z = 10e^{-j(\pi/2)}$ (d) $z = 10 \angle (-11\pi)$

Give numerical values for the real and imaginary parts.

PROBLEM 1.3:
Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = 2 - j2$ and $z_2 = 3e^{j3\pi/4}$.

(a) Conjugate: z_1^* (d) z_2^2 (g) $z_1 + z_2^*$

(b) jz_2 (e) $z_1^{-1} = 1/z_1$ (h) $|z_2|^2 = z_2 z_2^*$

(c) z_2/z_1 (f) $z_1 z_2$ (i) $z_2 + z_2^*$

Note: z^* means the "conjugate" of z. Part (h) is the *magnitude-squared.*

PROBLEM 1.4:
Simplify the following complex-valued expressions. Give your answer in either rectangular or polar form, whichever is most convenient. In parts (a)-(d) assume that A, α, and ϕ are positive real numbers. Your answers to parts (a)-(d) will be in terms of these quantities.

(a) For $z = Ae^{-j\pi/3}$, determine a simple expression for $\Re e\{z^*\}$.

(b) For $z = Ae^{-j\pi/3}$, determine a simple expression for $z - z^*$.

(c) For $z = 10e^{j\phi}$, determine a simple expression for $\Im m\{jz\}$.

(d) For $z = \alpha - j\alpha$, determine a simple expression for z in polar form.

(e) For $z = Ae^{-j\pi/3}$, determine a simple expression for $|z|/z$ in polar form.

PROBLEM 1.5*:
Simplify the following and give the answer in polar form. Make a plot of all the vectors involved in the complex addition.
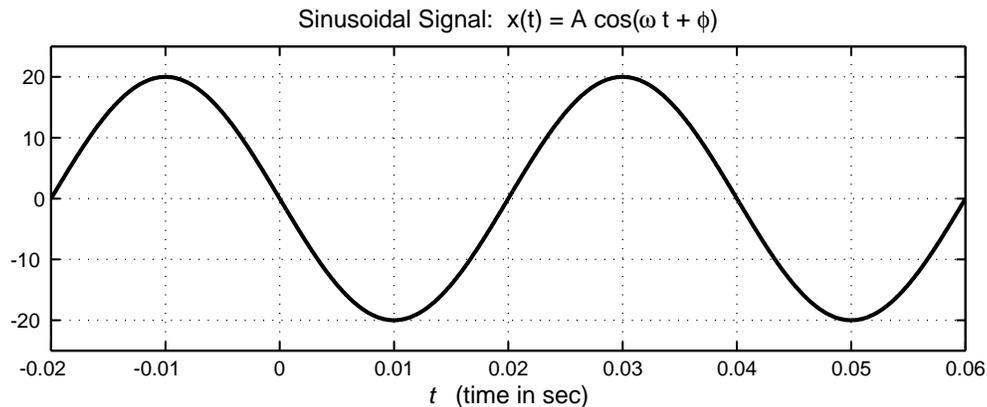
(a) $z_a = e^{-j2\pi/3} + e^{j2\pi/3}$

(b) $z_b = 1 + e^{j(\pi/2)} + \sqrt{2}e^{-j(3\pi/4)}$

(c) In addition, write the MATLAB statements that will perform the addition and also display the magnitude and phase of the result. Consult help on the MATLAB functions abs and angle, and also the DSP-First Toolbox functions: zprint, zvect, etc. Use these to check your hand calculations.

PROBLEM 1.6*:

The waveform in the following figure can be expressed as

$$x(t) = A \cos[\omega_0(t - t_m)] = A \cos(\omega_0 t + \phi) = A \cos(2\pi f_0 t + \phi)$$

From the waveform, determine A, ω_0, f_0, t_m, and ϕ. Choose the value of ϕ such that $-\pi < \phi \le \pi$.

Sinusoidal Signal: $x(t) = A \cos(\omega t + \phi)$

PROBLEM 1.7*:

Suppose that MATLAB is used to plot a sinusoidal signal. The following MATLAB code generates the signal and makes the plot. Draw a sketch of the plot that will be done by MATLAB. Determine the amplitude (A), phase (ϕ), and period of the sinusoid and label the period on your plot.

```
dt = 0.001;
tt = -.02 : dt : .06;
Fo = 25;
Z =10*sqrt(2)*(1+j);
xx = real( Z*exp( 2j*pi*Fo*tt ) );
%
plot( tt, xx ),   grid
title( 'SECTION of a SINUSOID' ),    xlabel('TIME   (sec)')
```

PROBLEM 1.8*:

Sinusoidal Signal: x(t) = A cos(ω t + φ)

Answer to part (c): y(t) = 6 x(t+0.1)
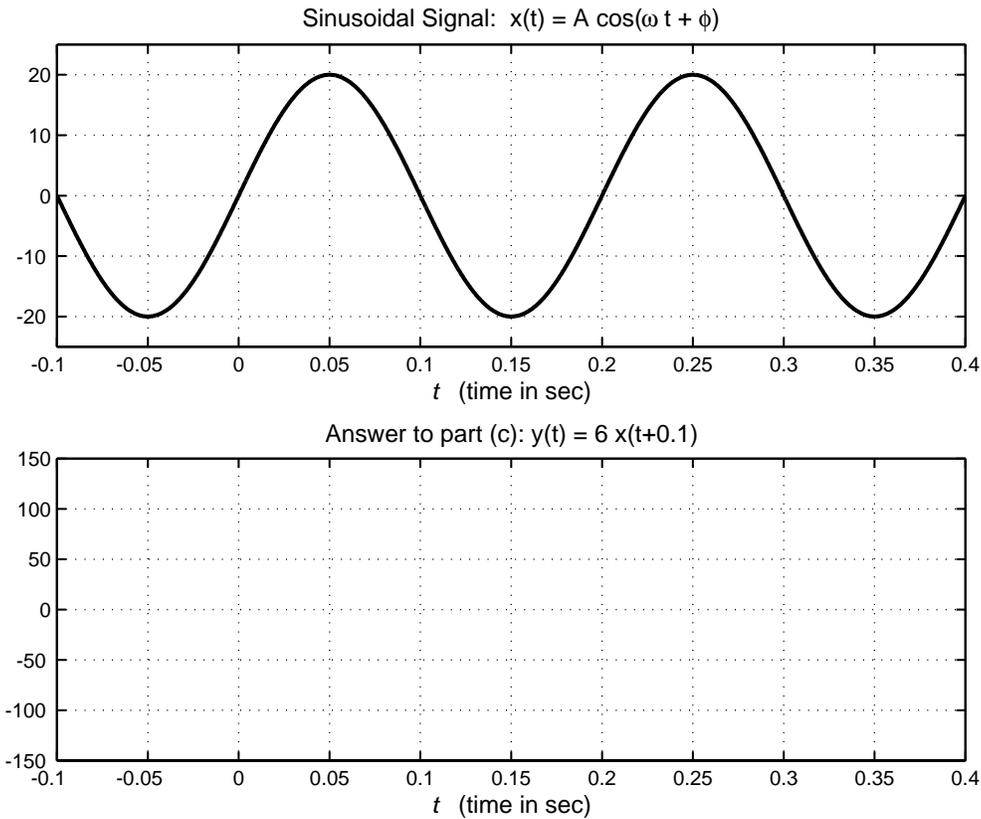
(a) The above figure shows a plot of a sinusoidal wave $x(t)$. From the plot, determine the values of A, ω_0, and $-\pi < \phi \leq \pi$ in the representation

$$x(t) = A\cos(\omega_0 t + \phi)$$

Where appropriate, be sure to indicate the units of the sinusoidal signal parameters.

(b) The signal $x(t)$ in part (a) can be written as the real part of a complex exponential. Determine Z for the complex signal $z(t) = Ze^{j\omega_0 t}$ such that $x(t) = \Re e\{z(t)\}$.

(c) Sketch the signal $y(t) = 6x(t+0.1)$, where $x(t)$ is the signal from part (a). Use the axes provided above or make your own axes covering the same time interval.

PROBLEM 1.9*:

Simplify the following and give the answer as a single sinusoid: $x(t) = A\cos(\omega t + \phi)$. Draw the vector diagram of the complex amplitudes (phasors) to show how you obtained the answer.

(a) $x_a(t) = 2\cos(333\pi t) - \sin(333\pi t)$

(b) $x_b(t) = 10\cos(245t + 3\pi/4) + 10\cos(245t + \pi/2)$

(c) $x_c(t) = \cos(41t + 17\pi) + \sqrt{2}\cos(41t + \pi/4) + \sqrt{2}\cos(41t - \pi/4)$