Georgia Institute of Technology

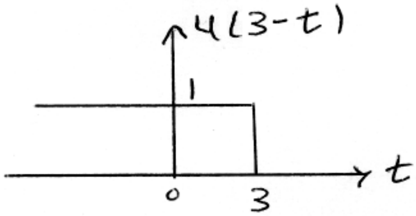
School of ECE

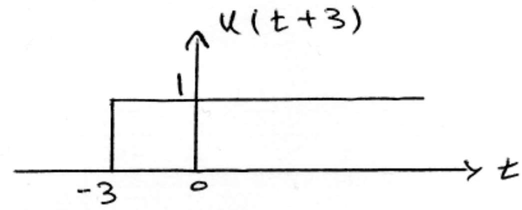
ECE 2025 Fall 2000

HW 11 Solutions

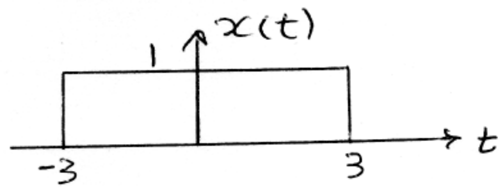
11.1

(a)





$x(t) = u(t+3) \cdot u(3-t) \longrightarrow$



$T_0/2 = 3 \quad \rightarrow \quad X(j\omega) = \dfrac{\sin(3\omega)}{\omega/2}$

(b) Note

t-domain $\quad\quad$ ω-domain

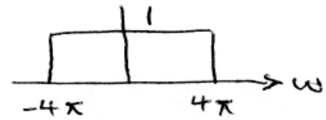
$\sin 4\pi t \longleftrightarrow \frac{\pi}{j}\delta(\omega - 4\pi) - \frac{\pi}{j}\delta(\omega + 4\pi)$

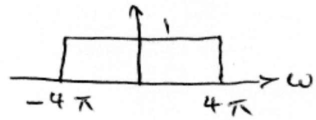
Convolution property of the Fourier Transform:

$$X(j\omega) = \frac{1}{2\pi}\left\{\frac{\pi}{j}\delta(\omega-4\pi) - \frac{\pi}{j}\delta(\omega+4\pi)\right\} * \left\{\frac{\pi}{j}\delta(\omega-50\pi) - \frac{\pi}{j}\delta(\omega+50\pi)\right\}$$

$$X(j\omega) = \frac{\pi}{2}\delta(\omega-46\pi) + \frac{\pi}{2}\delta(\omega+46\pi) - \frac{\pi}{2}\delta(\omega-54\pi) - \frac{\pi}{2}\delta(\omega+54\pi)$$

11.1

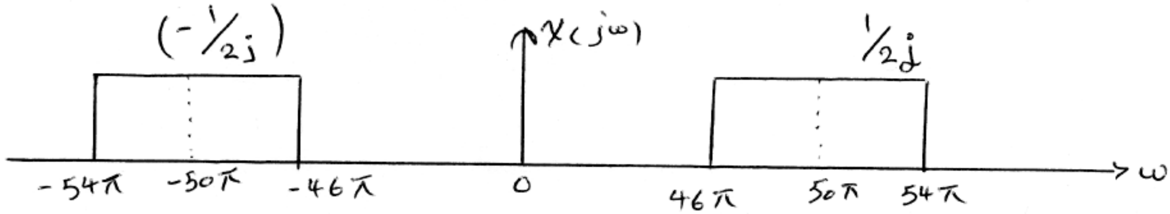
(c) Note $\dfrac{\sin 4\pi t}{\pi t} \longleftrightarrow$ 

$X(j\omega) = \dfrac{1}{2\pi}\Big\{$ 

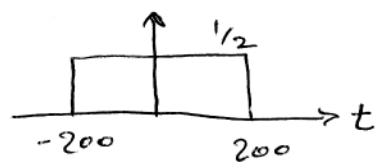
$\Big\} * \Big\{ \dfrac{\pi}{j}\,\delta(\omega - 50\pi) - \dfrac{\pi}{j}\,\delta(\omega + 50\pi) \Big\}$

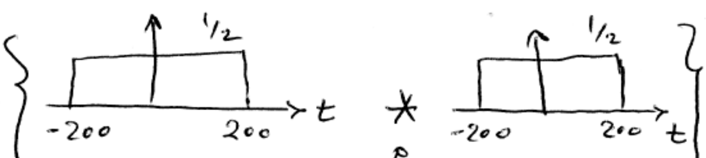
convolution

Thus



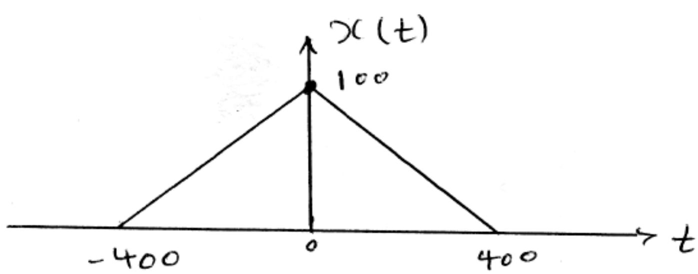
$$X(j\omega) = \begin{cases} \dfrac{1}{2j} & 46\pi \le \omega \le 54\pi \\ -1/2j & -54\pi \le \omega \le -46\pi \\ 0 & \text{else} \end{cases}$$

(d) Note: $\dfrac{1}{2}\,\dfrac{\sin(200\omega)}{\omega/2} \longleftrightarrow$ 

$\dfrac{\sin^2(200\omega)}{\omega^2} \longleftrightarrow$ 

convolution

Thus



11.1

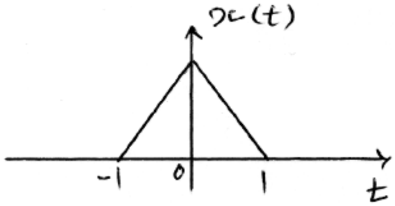
(e) $\cos\omega \longleftrightarrow \frac{1}{2}\{\delta(t-1)+\delta(t+1)\}$

$\cos^2\omega \longleftrightarrow \frac{1}{2}\{\delta(t-1)+\delta(t+1)\} * \frac{1}{2}\{\delta(t-1)+\delta(t+1)\}$

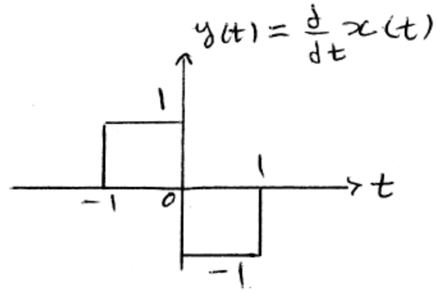
$x(t)=\frac{1}{4}\{\delta(t-2)+\delta(t+2)+2\delta(t)\}$

11.2

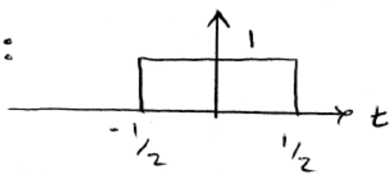
(a)



$\Rightarrow$

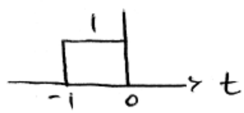


since:

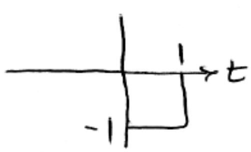


$\longleftrightarrow \dfrac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$

then



$\longleftrightarrow e^{j\frac{\omega}{2}}\dfrac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$



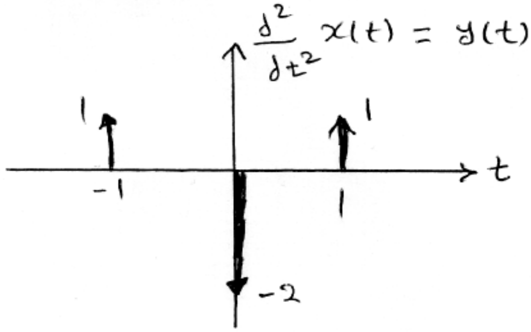
$\longleftrightarrow (-1)e^{-j\frac{\omega}{2}}\dfrac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$

Thus: $Y(j\omega)=\dfrac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}\left(e^{-j\frac{\omega}{2}}-e^{-j\frac{\omega}{2}}\right)=\dfrac{4j}{\omega}\sin^2\left(\frac{\omega}{2}\right)$

$Y(j\omega)=j\omega\, X(j\omega) \Longrightarrow X(j\omega)=\dfrac{4}{\omega^2}\sin^2\dfrac{\omega}{2}=\left(\dfrac{\sin\frac{\omega}{2}}{\omega/2}\right)^2$

11.2

(b)

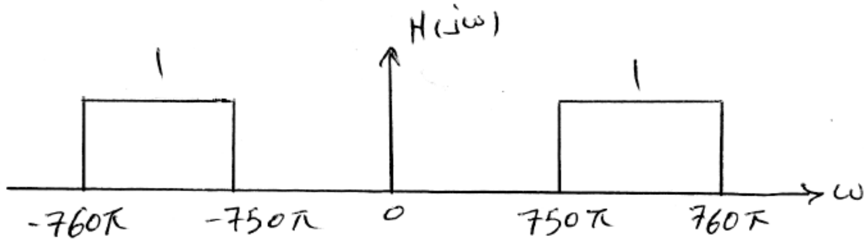


$$Y(j\omega) = e^{j\omega} + e^{-j\omega} - 2 = 2\cos\omega - 2 = -4\sin^2\left(\frac{\omega}{2}\right)$$

$$X(j\omega) = \left(\frac{1}{j\omega}\right)^2 Y(j\omega) = \frac{4}{\omega^2}\sin^2\left(\frac{\omega}{2}\right)$$

11.3

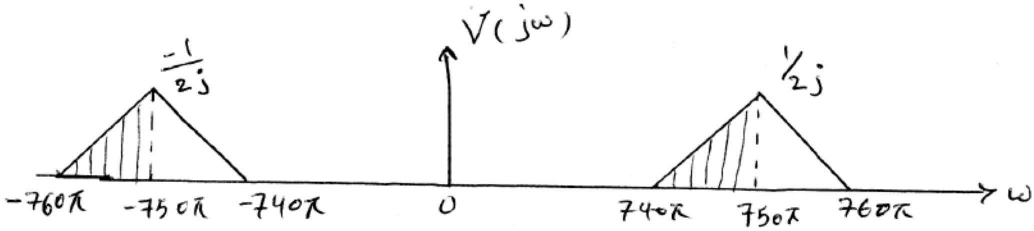
(a)



(b) $v(t) = x(t) \cdot \sin(750\pi t)$

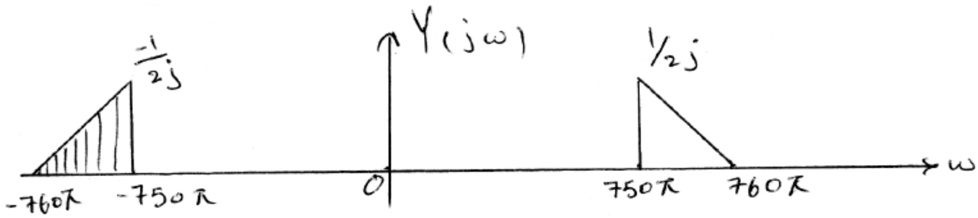
$$V(j\omega) = \frac{1}{2\pi} X(j\omega) * \left\{ \frac{\pi}{j}\delta(\omega - 750\pi) - \frac{\pi}{j}\delta(\omega + 750\pi) \right\}$$

$$V(j\omega) = \frac{1}{2j} X(j(\omega - 750\pi)) - \frac{1}{2j} X(j(\omega + 750\pi))$$



11.3

(c) $Y(j\omega) = V(j\omega) \cdot H(j\omega)$



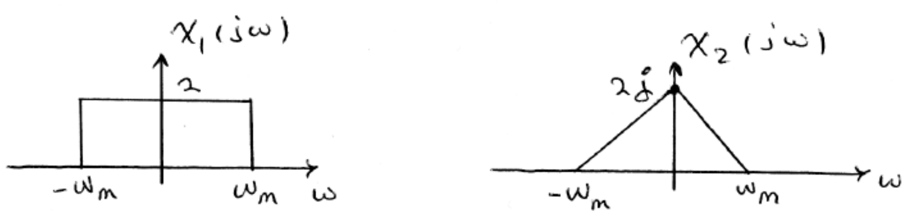
11.4

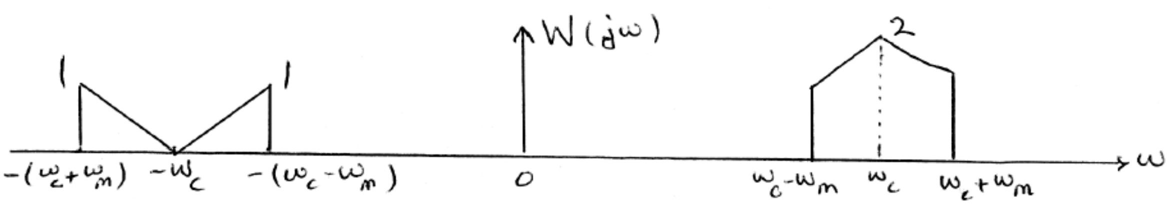
$$w(t) = x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t$$

(a) $$W(j\omega) = \frac{1}{2\pi} X_1(j\omega) * \left\{ \pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c) \right\} + \frac{1}{2\pi} X_2(j\omega) * \left\{ \frac{\pi}{j}\delta(\omega-\omega_c) - \frac{\pi}{j}\delta(\omega+\omega_c) \right\}$$

$$W(j\omega) = \frac{1}{2} X_1(j(\omega-\omega_c)) + \frac{1}{2} X_1(j(\omega+\omega_c)) + + \frac{1}{2j} X_2(j(\omega-\omega_c)) - \frac{1}{2j} X_2(j(\omega+\omega_c))$$

Assume :





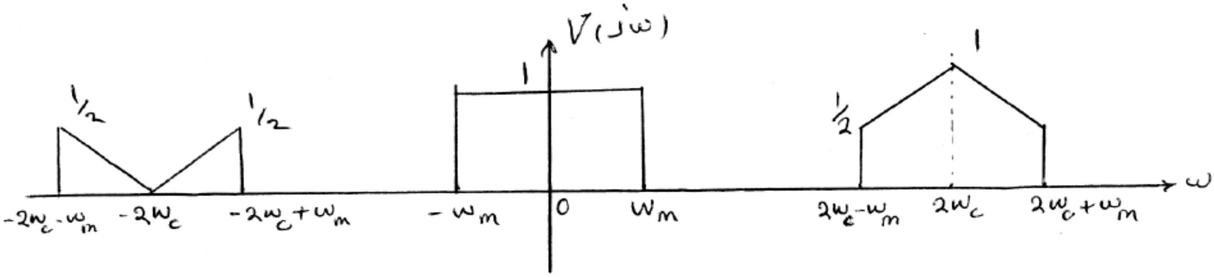
(b) $\omega_a = \omega_c - \omega_m \quad , \quad \omega_b = \omega_c + \omega_m$

(c) $V(t) = W(t)\cos\omega_c t = x_1(t)\cos^2\omega_c t + x_2(t)\sin\omega_c t \cdot \cos\omega_c t$

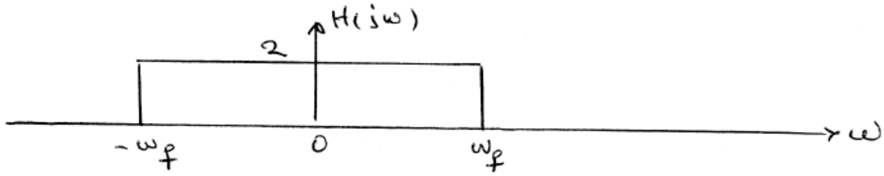
$$V(t) = \frac{1}{2}x_1(t)\left(1+\cos 2\omega_c t\right) + \frac{1}{2}x_2(t)\cdot\left(\sin 2\omega_c t\right)$$

(d) $V(t) = \frac{1}{2}x_1(t) + \frac{1}{2}x_1(t)\cos 2\omega_c t + \frac{1}{2}x_2(t)\sin 2\omega_c t$

If we use the plot in part (a), we have:

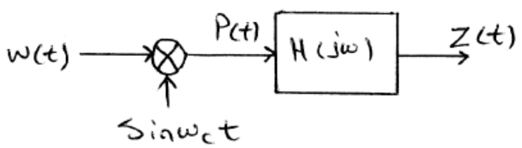


From the above plot, it is clear that we should have:



$$H(j\omega) = \begin{cases} 2 & |\omega| \leq \omega_f \\ 0 & \text{else} \end{cases} \quad \text{where} \quad \omega_m \leq \omega_f \leq 2\omega_c - \omega_m$$

(e)



$$P(t) = x_1(t)\sin\omega_c t\cdot\cos\omega_c t + x_2(t)\sin^2\omega_c t$$

$$p(t) = \frac{1}{2}x_1(t)\sin 2\omega_c t + x_2(t)\,\frac{1-\cos 2\omega_c t}{2}$$

with similar argument, $H(j\omega)$ is the same as part (d).

11.5

(a) $\omega_s \geq 2(80\pi)$

$x_s(t) = x(t) \cdot P(t) \qquad X_s(j\omega) = \frac{1}{2\pi}\{X(j\omega) * P(j\omega)\}$

$$P(j\omega) = \frac{1}{T_s}\sum_{K=-\infty}^{+\infty} \delta(\omega - K\omega_s) \quad \text{where} \quad \omega_s = \frac{2\pi}{T_s}$$

Thus, $X_s(j\omega) = \frac{1}{2\pi T_s}\sum_{K} X(j(\omega - K\omega_s))$

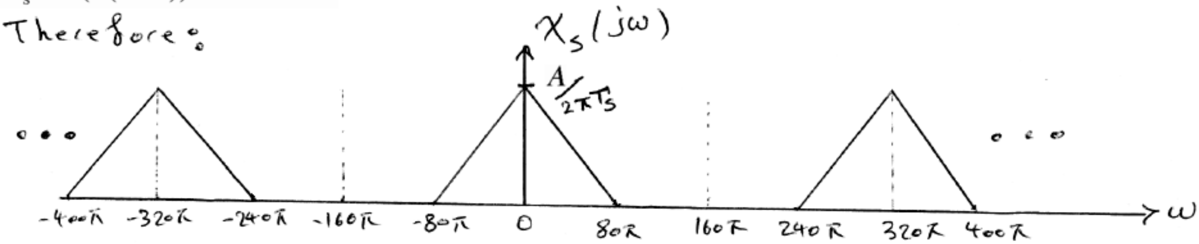
The above expression is obtained using the fact that

$$X(j\omega) * \delta(\omega - K\omega_s) = X(j(\omega - K\omega_s))$$

For twice Nyquist we need

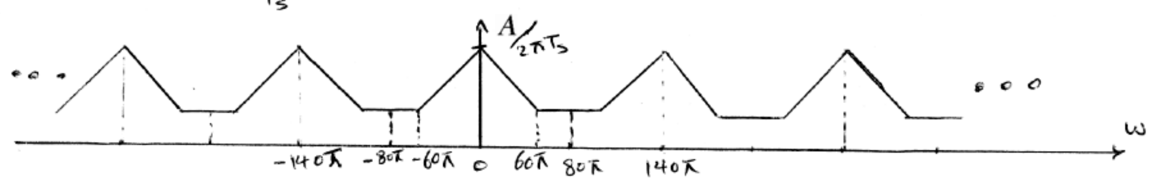
$\omega_s = 2(2(80\pi)) = 320\pi$

Therefore:



(b)

since $\omega_s = \frac{2\pi}{T_s} = 140\pi$, there is aliasing:



(c)