HOMEWORK #10
SOLUTIONS

10.1 a) $$H(j\omega) = \int_{-\infty}^{+\infty} [\delta(t) + 5e^{-5t}u(t)]e^{-j\omega t}\,dt =$$

$$= \int_{-\infty}^{+\infty} \delta(t) e^{-j\omega t}\,dt + 5\int_{0}^{+\infty} e^{-(5+j\omega)t}\,dt =$$

$$= 1 + 5\left[\frac{-e^{-(5+j\omega)t}}{5+j\omega}\right]_0^{+\infty} = 1 + 5\,\frac{1}{5+j\omega} =$$

$$= \frac{10+j\omega}{5+j\omega}$$

b) See next page.

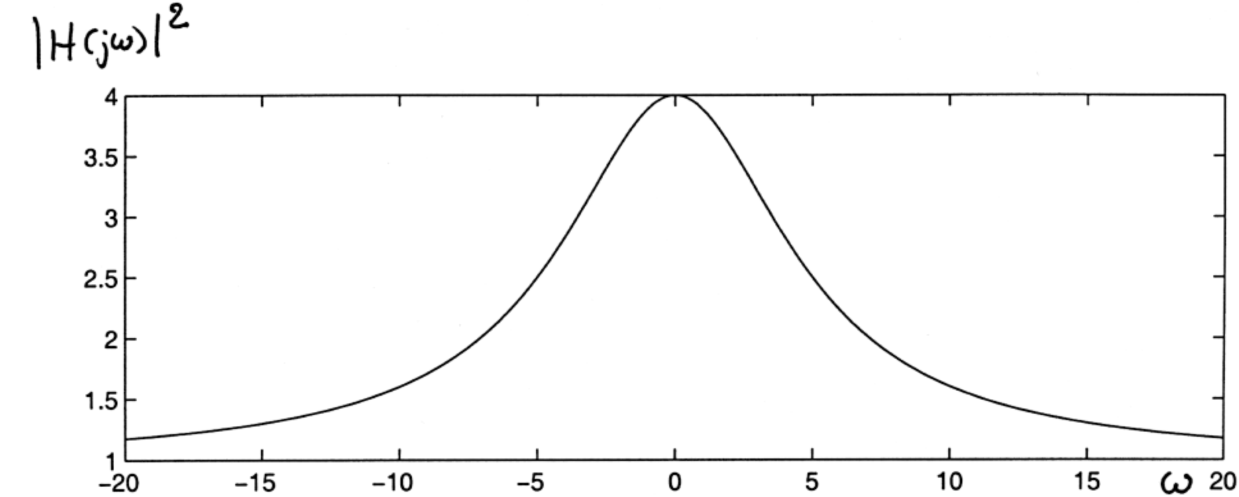
c) $|H(j\omega)|^2 = \max$ at $\omega = 0$; $|H(j0)|^2 = 4$.

To compute the 3-dB point:

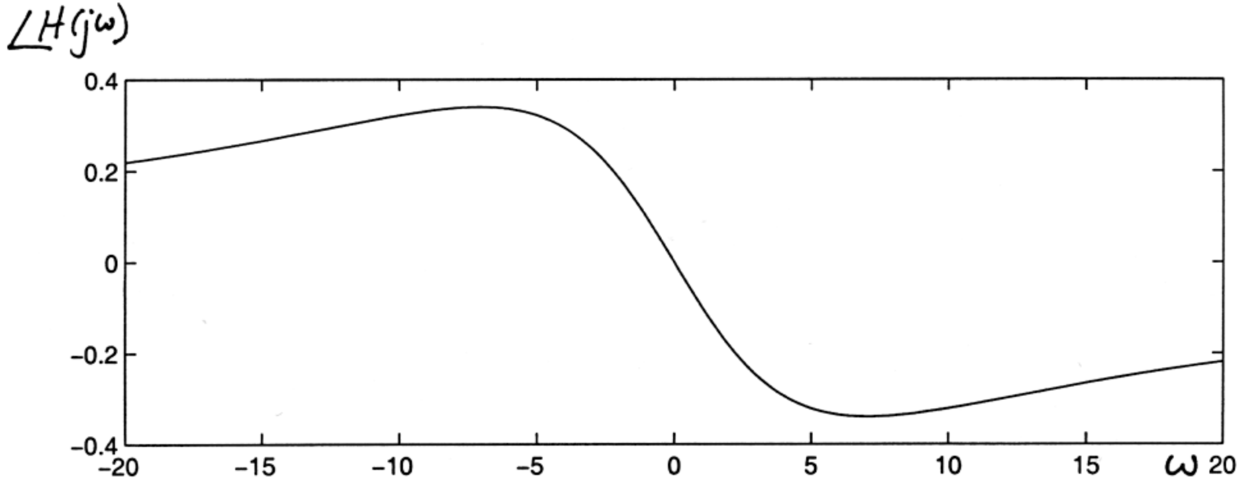
$$|H(j\omega)|^2 = \frac{100+\omega^2}{25+\omega^2} = \frac{1}{2}\cdot 4 = 2$$

$$100+\omega^2 = 50 + 2\omega^2 \Rightarrow \omega^2 = 50 \Rightarrow \omega = \sqrt{50} =$$

$$\cong 7.07 \text{ rad/sec.}$$



|H(jω)|²
4
3.5
3
2.5
2
1.5
1
−20
−15
−10
−5
0
5
10
15
ω
20



∠H(jω)
0.4
0.2
0
−0.2
−0.4
−20
−15
−10
−5
0
5
10
15
ω
20

10.1 d) Consider each term separately:

- $x(t) = 1$

  $y(t) = H(j0) \cdot 1 = 2$

- $x(t) = 2\cos(100t)$

  $$H(j100) = \frac{10 + j100}{5 + j100} = 1.004\, e^{-j\,0.05}$$

  $$y(t) = 2 \cdot 1.004 \cos(100t - 0.05)$$

- $x(t) = \delta(t-1)$

  $$y(t) = h(t) * x(t) = h(t) * \delta(t-1) =$$
  $$= h(t-1) = \delta(t-1) + 5e^{-5(t-1)}u(t-1)$$

Now combine the outputs:

$$y(t) = 2 + 2.008\cos(100t - 0.05) +$$
$$+ \delta(t-1) + 5e^{-5(t-1)}u(t-1).$$

10.2. a) $$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,dt = \frac{1}{16}\int_{-2}^{2} 1\,dt = \frac{1}{4}$$

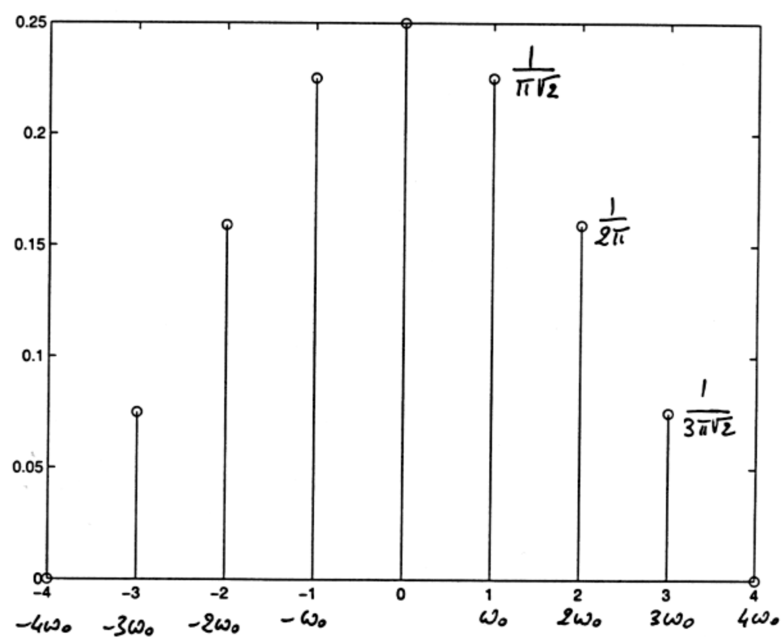
For $k \neq 0$: $$a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,e^{-jk\omega_0 t}\,dt =$$

$$= \frac{1}{16}\int_{-2}^{2} e^{-jk\omega_0 t}\,dt = \frac{1}{16}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-2}^{2} =$$

$$= \frac{1}{16}\;\frac{e^{-jk2\omega_0} - e^{jk2\omega_0}}{-jk\omega_0} = \frac{1}{8k\omega_0}\sin 2k\omega_0 =$$

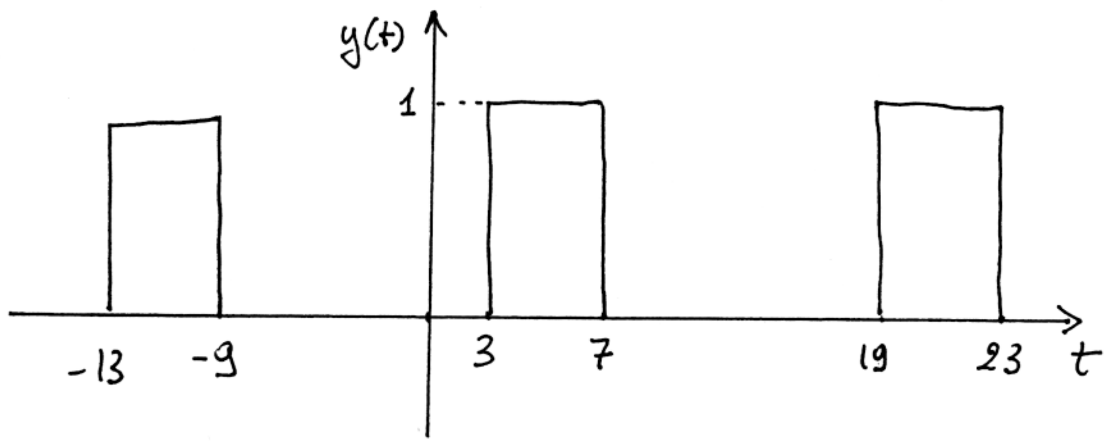
$$= \frac{1}{\pi k}\sin k\frac{\pi}{4}$$

b)

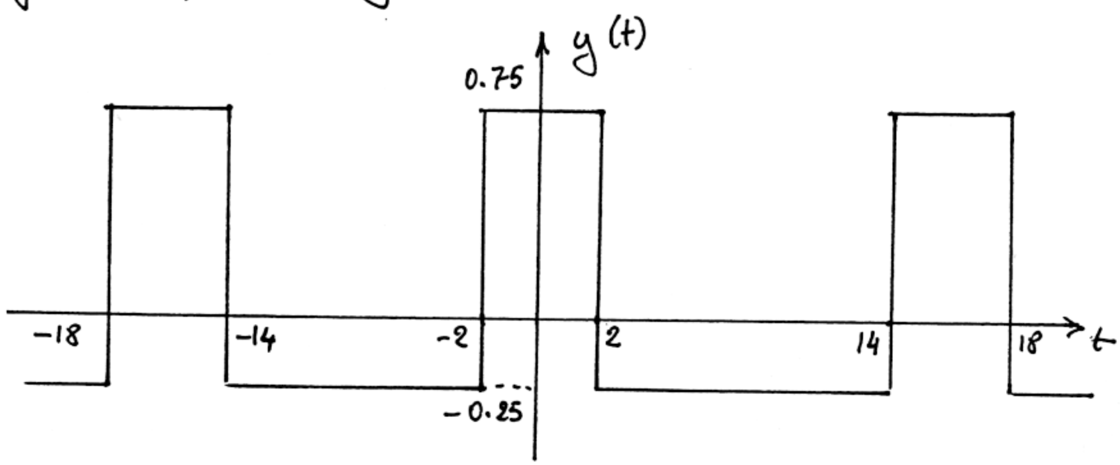


c) $H(j\omega) = e^{-j5\omega} \Longleftrightarrow h(t) = \delta(t-5)$

$y(t) = h(t) * x(t) = \delta(t-5) * x(t) =$
$x(t-5)$



d) This high-pass filter blocks only the DC component of $x(t)$, so $y(t) = x(t) - 0.25$



e) The low-pass filter must eliminate all harmonics except the DC component and the first harmonic. The (angular) frequency of the first harmonic is $\omega_0 = \frac{2\pi}{16} = \frac{\pi}{8}$ rad/sec. The frequency of the second harmonic is $2\omega_0 = \frac{\pi}{4}$ rad/sec. So any value of ω_c such that: $\frac{\pi}{8} < \omega_c < \frac{\pi}{4}$ will give the output $y(t) = \frac{1}{4} + \frac{\sqrt{2}}{\pi}\cos(\omega_0 t)$.

10.3. a) $h(t) = \delta(t) + 0.2\,\delta(t-2) + 0.8\,\delta(t-6)$

b) $$H(j\omega) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt + 0.2 \int_{-\infty}^{+\infty} \delta(t-2)\, e^{-j\omega t}\, dt +$$

$$+ 0.8 \int_{-\infty}^{+\infty} \delta(t-6)\, e^{-j\omega t}\, dt = 1 + 0.2\, e^{-j2\omega} + 0.8\, e^{-j6\omega}$$

c) $$y(t) = e^{j\omega t} + 0.2\, e^{j\omega(t-2)} + 0.8\, e^{j\omega(t-6)} =$$

$$= \left(1 + 0.2\, e^{-j2\omega} + 0.8\, e^{-j6\omega}\right) e^{j\omega t} = H(j\omega)\, e^{j\omega t}$$

10.4.a) $\frac{10}{1+0.5j\omega} = \frac{20}{2+j\omega} \Longleftrightarrow x(t) = 20e^{-2t}u(t)$

So: $\frac{10}{1+0.5j\omega}e^{-2j\omega} \Longleftrightarrow x(t) = 20e^{-2(t-2)}u(t-2)$

b) $5\cos(100\omega) = \frac{5}{2}e^{j100\omega} + \frac{5}{2}e^{-j100\omega} \Longleftrightarrow$

$x(t) = \frac{5}{2}\delta(t+100) + \frac{5}{2}\delta(t-100)$

c) $\delta(\omega-100\pi) + \delta(\omega+100\pi) \Longleftrightarrow$

$x(t) = \frac{1}{2\pi}e^{j100\pi t} + \frac{1}{2\pi}e^{-j100\pi t} =$

$= \frac{1}{\pi}\cos(100\pi t)$

10.5.a) $\delta(t-10) \Longleftrightarrow \quad X(j\omega) = e^{-j10\omega}$

b) $\frac{\sin 5\pi t}{\pi t} \Longleftrightarrow X(j\omega) = [u(\omega+5\pi) - u(\omega-5\pi)]$

So: $\frac{\sin[5\pi(t-10)]}{\pi(t-10)} \Longleftrightarrow X(j\omega) = e^{-j10\omega}[u(\omega+5\pi) + - u(\omega-5\pi)]$

c) $e^{-4t}u(t) \Leftrightarrow X(j\omega) = \dfrac{1}{4+j\omega}$

So: $e^{-4t}u(t) - e^{-40}e^{-4(t-10)}u(t-10) =$

$$= \frac{1}{4+j\omega} - e^{-40}\, e^{-j10\omega}\, \frac{1}{4+j\omega} =$$

$$= \frac{1 - e^{-(40+j10\omega)}}{4+j\omega}.$$