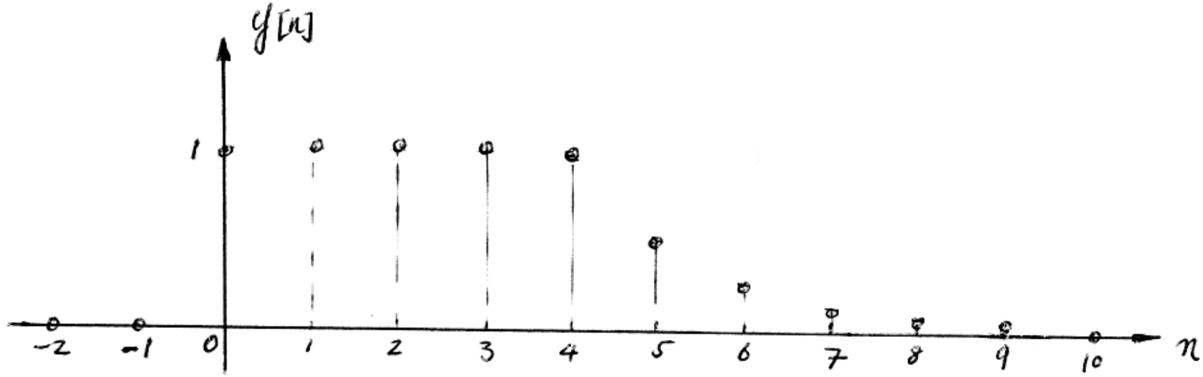
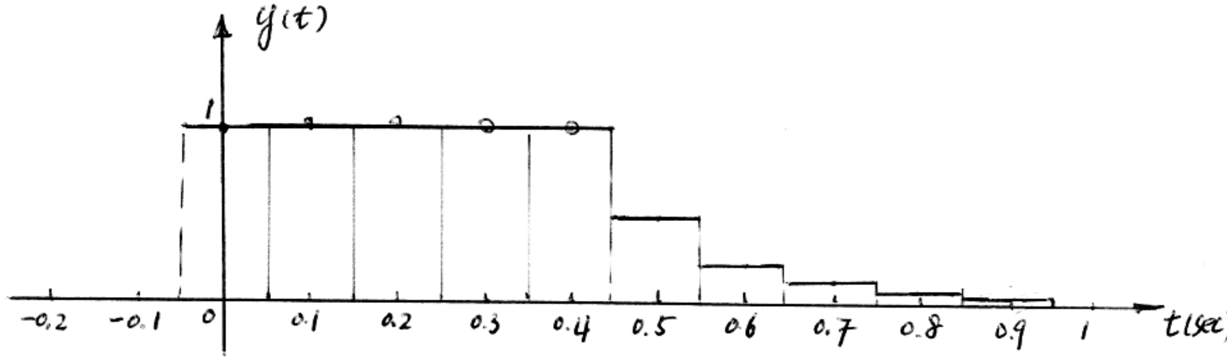
Problem 5.1

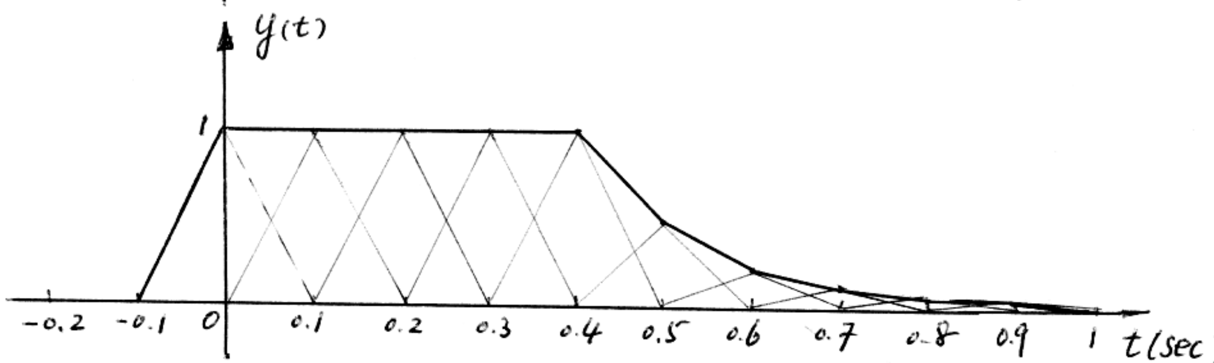
(a)



(b)



(c)



Problem 5.2

(a)

$$f_s = 1000 \text{ Hz}$$

$$\begin{aligned} x[n] &= x(n/f_s) \\ &= 7\cos(1800\pi \cdot n/1000 + \pi/4) \\ &= 7\cos(1.8\pi n + \pi/4) \\ &= 7\cos(1.8\pi n - 2\pi n + \pi/4) \\ &= 7\cos(-0.2\pi n + \pi/4) \\ &= 7\cos(0.2\pi n - \pi/4) \end{aligned}$$

$$\begin{aligned} y(t) &= x[t f_s] \\ &= 7\cos(0.2\pi \cdot t \cdot 1000 - \pi/4) \\ &= 7\cos(200\pi t - \pi/4) \end{aligned}$$

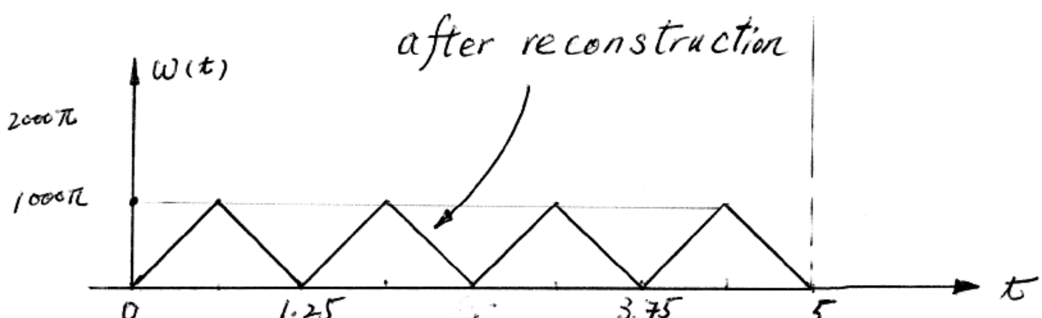
(b)

$\cos(2000\pi t - 400\pi t^2)$

$\Rightarrow \psi(t) = 2000\pi t - 400\pi t^2$

$\omega_i(t) = \frac{d}{dt}\psi(t) = 2000\pi - 800\pi t$

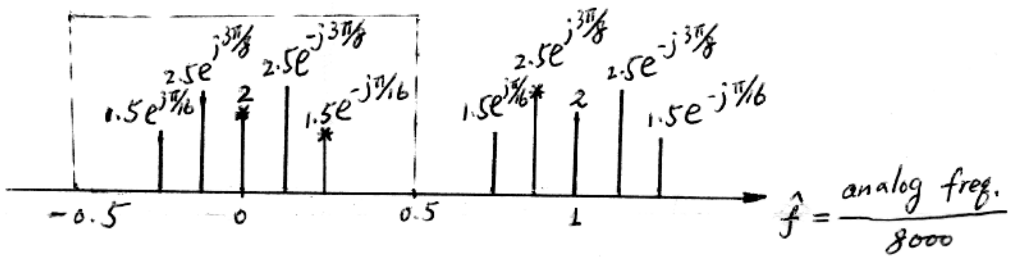
With a sampling frequency of $f_s = 1000$ Hz, the highest frequency reconstructed by the D / A converter would be $\omega = 2\pi(500)$ rad / sec. For example at $t = 4$, $\omega_i(4) = 2000\pi - 3200\pi = -1200\pi$ rad / sec. which aliases to $\omega = 2\pi(-1200 + 1000) = -2\pi(200)$ rad / sec. Since a cosine with negative frequency is the same as positive frequency, the output frequency is $2\pi(200)$ rad / sec.



Problem 5.3

(a)

$$
\begin{aligned}
x[n] &= x(n/f_s) \\
&= 2 + 3\cos(2\pi(2000)n/8000 - \pi/16) \\
&\quad + 5\cos(2\pi(7000)n/8000 + 3\pi/8) \\
&= 2 + 3\cos(0.5\pi n - \pi/16) \\
&\quad + 5\cos(1.75\pi n + 3\pi/8) \\
&= 2 + 3\cos(0.5\pi n - \pi/16) \\
&\quad + 5\cos(1.75\pi n - 2\pi n + 3\pi/8) \\
&= 2 + 3\cos(0.5\pi n - \pi/16) \\
&\quad + 5\cos(-0.25\pi n + 3\pi/8) \\
&= 2 + 3\cos(0.5\pi n - \pi/16) \\
&\quad + 5\cos(0.25\pi n - 3\pi/8)
\end{aligned}
$$



(b)

$\hat{f} = 0,\ 0.125,\ \text{and } 0.25$, $f_s = 8000$

$f = \hat{f} f_s = 0,\ 1000\text{ Hz},\ \text{and } 2000\text{ Hz}$

(c)

$f_s = 16000$

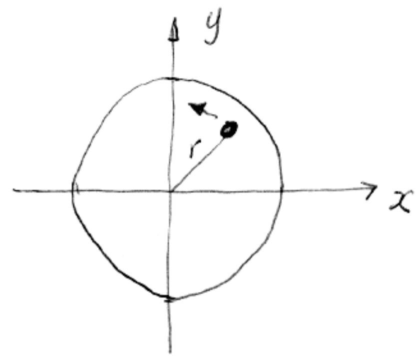
$f = \hat{f} f_s = 0,\ 2000\text{ Hz},\ \text{and } 4000\text{ Hz}$

Problem 5.4

(a) $z(t) = x(t) + j\,y(t)$

$= r e^{j 2\pi (1.5) t}$

($\phi = 0$ assume the initial phase is zero)



(b) $T_s = 50 \text{ msec} = 0.05 \text{ sec}$

$f_s = 20 \text{ Hz}$

$z[n] = z(n/f_s)$

$= r e^{j 2\pi (1.5) \cdot n/20}$

$= r e^{j 1.5 \pi n}$

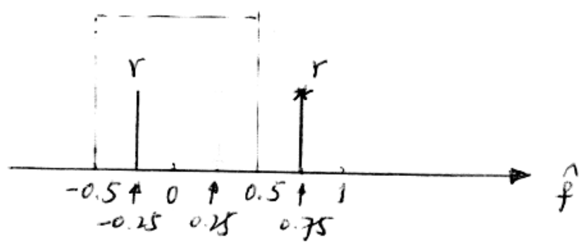
$= r e^{-j 2\pi n + j 1.5 \pi n}$

$= r e^{-j 0.5 \pi n}$

$0.25 \times 20 = 5 \implies$ the spot rotates at 5 revolutions/sec

"$-$" sign $\implies$ the rotation is in the <u>clockwise</u> direction

(c)



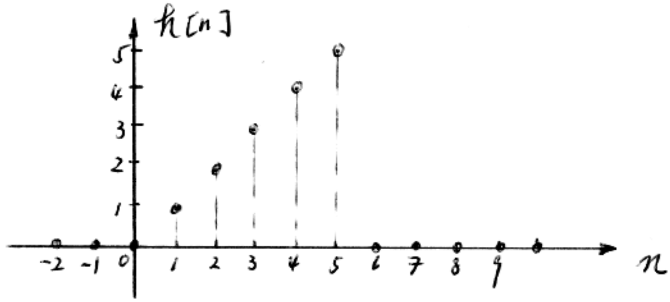
Problem 5.5

(a)

$$b_k = \begin{cases} k & k = 0, 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

(b)

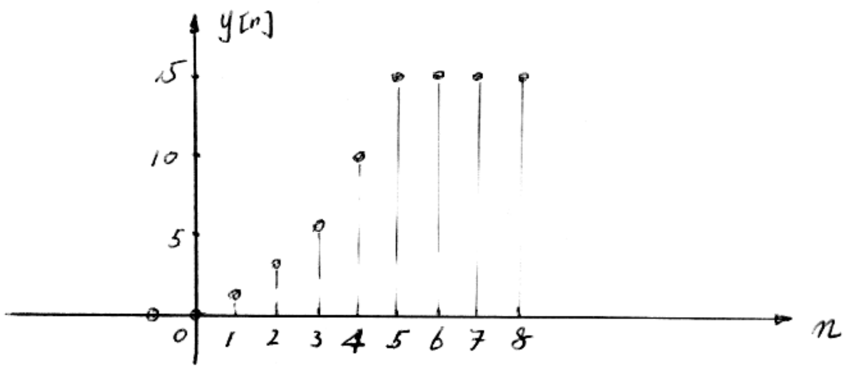
$$h[n] = b_n = \begin{cases} n & n = 0, 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$



(c)

$$y[n] = \sum_{k=0}^{5} k\,x[n-k] = \sum_{k=0}^{\max(5,n)} k \implies$$

n	<0	0	1	2	3	4	5	>5
y[n]	0	0	1	3	6	10	15	15



Problem 5.6

$y[n] = \sum_{k=0}^{20} b_k x[n-k]$ is guaranteed to be zero

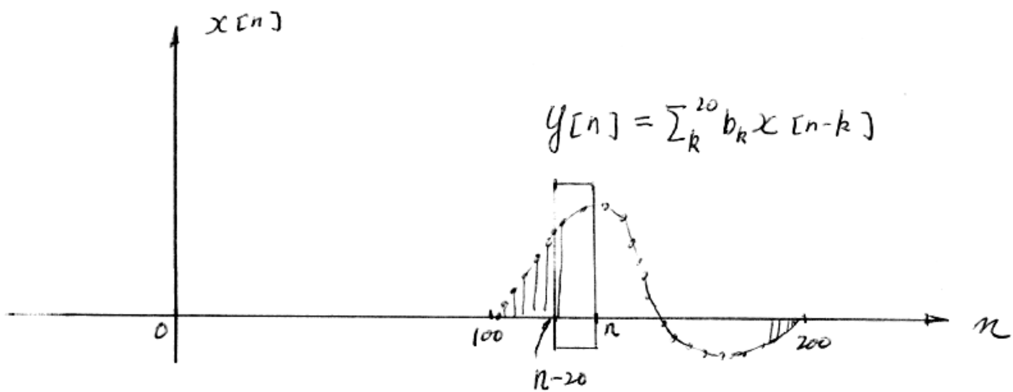
$\Rightarrow \quad x[n-k] = 0 \quad$ for $k = 0, 1, \cdots, 20$

(a) $\quad n - k < 100 \quad$ for $k = 0, 1, \cdots, 20$

$n < 100 \quad \Longrightarrow \quad N_1 = 100$

(b) $\quad n - k > 200 \quad$ for $k = 0, 1, \cdots, 20$

$n > 200 + 20 \quad \Longrightarrow \quad N_2 = 220$



Problem 5.7

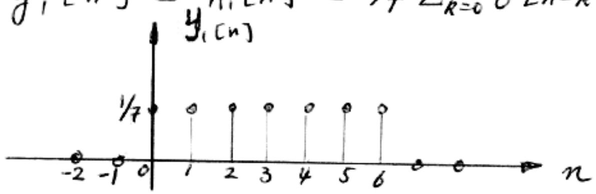
(a) For System #1,

$$b_n = \begin{cases} 0 & n<0 \\ 1/7 & n=0,1,2,3,4,5,6 \\ 0 & n>0 \end{cases}$$

For System #2,

$$b_n = \begin{cases} 1 & n=0 \\ -1 & n=1 \\ 0 & \text{otherwise} \end{cases}$$

(b) $y_1[n] = h_1[n] = 1/7 \sum_{k=0}^{6} \delta[n-k]$



(c)

$$\begin{aligned} y[n] &= y_1[n] - y_1[n-1] \\ &= 1/7 \sum_{k=0}^{6} \delta[n-k] - 1/7 \sum_{k=0}^{6} \delta[n-1-k] \\ &= 1/7 \sum_{k=0}^{6} \delta[n-k] - 1/7 \sum_{k=1}^{7} \delta[n-k] \\ &= 1/7\, \delta[n] - 1/7\, \delta[n-7] \end{aligned}$$

Since $y[n]$ is the output of the cascade system when input is a impulse $\delta[n]$, the impulse response of the system is

$$h[n] = y[n] = \begin{cases} 1/7 & n=0 \\ -1/7 & n=7 \\ 0 & \text{otherwise} \end{cases}$$