ECE 2025   Fall 2000
Problem Set #5

Reading: Chapter 4 on *Sampling*, and Chapter 5, pp. 119-133.

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 5.1:

A non-ideal D-to-C converter takes a sequence $y[n]$ as input and produces a continuous-time output $y(t)$ according to the relation

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

where $T_s = 0.1$ second. The input sequence is given by the formula

$$y[n] = \begin{cases} 1 & 0 \le n \le 4 \\ (0.5)^{(n-4)} & 5 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$
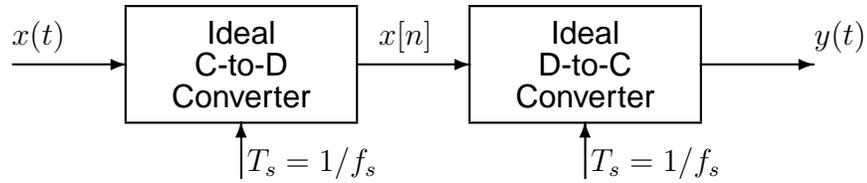
(a) Plot $y[n]$ versus n.

(b) For the pulse shape

$$p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

(c) For the pulse shape

$$p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$$

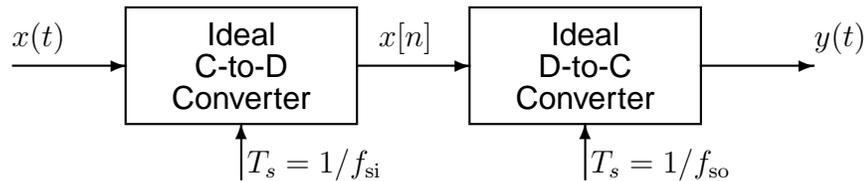carefully sketch the output waveform $y(t)$ over its non-zero region.

PROBLEM 5.2:

$x(t)$ → [Ideal C-to-D Converter] → $x[n]$ → [Ideal D-to-C Converter] → $y(t)$

$T_s = 1/f_s$ $T_s = 1/f_s$

Chirps are very useful signals for probing the behavior of sampling operations and illustrating the "folding" type of aliasing (see Fig. 4.4 in the book).

(a) If the input to the ideal C/D converter is $x(t) = 7\cos(1800\pi t + \pi/4)$, and the sampling frequency is 1000 Hz, then the output $y(t)$ is a sinusoid. Determine the formula for the output signal.

(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(2000\pi t - 400\pi t^2) \qquad \text{for } 0 \le t \le 5 \text{ sec.}$$

If the sampling rate is $f_s = 1000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`, or the DSP-First function `plotspec()`.

PROBLEM 5.3*:

$x(t)$ → [Ideal C-to-D Converter] → $x[n]$ → [Ideal D-to-C Converter] → $y(t)$

$T_s = 1/f_{si}$ $T_s = 1/f_{so}$

(a) Suppose that the input $x(t)$ is given by

$$x(t) = 2 + 3\cos(2\pi(2000)t - \pi/16) + 5\cos(2\pi(7000)t + 3\pi/8)$$

Determine the spectrum for $x[n]$ when $f_{si} = 8000$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, making sure to label the frequency, amplitude and phase of each spectral component.

(b) Using the discrete-time spectrum from part (a), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_{so} = 8000$ Hz.

(c) Again using the discrete-time spectrum from part (b), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_{so} = 16000$ Hz. In other words, the sampling rates of the two converters are different.

PROBLEM 5.4*:

In the rotating disk and strobe demo described in Chapter 4 of *DSP First,* we observed that different flashing rates of the strobe light would make the spot on the disk stand still.

(a) Assume that the disk is rotating in the counter-clockwise direction at a constant speed of 15 revolutions per second. Express the movement of the spot on the disk as a rotating complex phasor.

(b) Assume that the flashing rate is fixed so that the interval between flashes is 50 milliseconds. Explain how the spot will move and write a complex phasor that gives the position of the spot at each flash.

(c) Draw a spectrum plot of the discrete-time signal in part (b) to explain your answer.

PROBLEM 5.5*:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{5} kx[n-k]$$

The input to this system is *unit step* signal, denoted by $u[n]$, i.e., $\quad x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Find the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Find the output $y[n]$ when the input is $u[n]$, and make a plot of $y[n]$ vs. n. (Hint: you might find it useful to check your results with MATLAB's `conv()` function.)

PROBLEM 5.6*:

Consider a system defined by $\quad y[n] = \sum_{k=0}^{20} b_k x[n-k]$

Suppose that the input signal $x[n]$ is equal to zero for $n < 100$, and also for $n > 200$. Then it is possible to find regions of n where the output is guaranteed to be zero.

(a) Show that $y[n]$ is zero for $n < N_1$, and find the integer N_1 for which this is true. (If it is convenient assume that $x[n]$ is equal to one for $n = 100, 101, 102, 103, \ldots, 200$.)

(b) In addition, $y[n]$ will be zero for $n > N_2$. Find the integer N_2 for which this is true.

Hints: consult Figs. 5.5 and 5.6 in the book for the sliding window interpretation of the FIR filter. In addition, analyze the index $[n-k]$ when n is fixed and k varies over the summation range.

PROBLEM 5.7*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

$$x[n] \longrightarrow \boxed{\begin{array}{c} \text{LTI} \\ \text{System \#1} \\ h_1[n] \end{array}} \xrightarrow{y_1[n]} \boxed{\begin{array}{c} \text{LTI} \\ \text{System \#2} \\ h_2[n] \end{array}} \longrightarrow y[n]$$

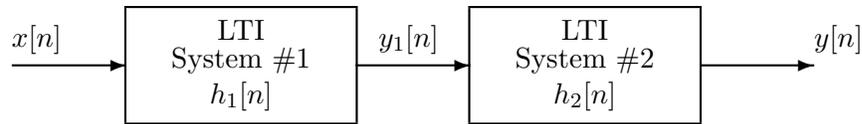Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a seven-point running average filter,

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 1/7 & n = 0, 1, 2, 3, 4, 5, 6 \\ 0 & n > 6 \end{cases}$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n] - y_1[n-1]$$

We can find the impulse response of the overall system, by first finding the impulse response of the first system and then use that response as the input to the second system.

(a) Determine the filter coefficients of System #1, and also for System #2.

(b) When the input signal $x[n]$ is an impulse, $\delta[n]$, determine the signal $y_1[n]$ and make a plot.

(c) Determine the impulse response of the overall cascade system, i.e., find $y[n]$ when $x[n] = \delta[n]$. Take advantage of the fact that you already know the output of System #1, $y_1[n]$, when its input is an impulse.