

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2000
Problem Set #12

Assigned: 8-April-00

Due Date: Week of 17-April-00

Final Exam is scheduled for Tuesday, 2-May, at 11:30 AM. Coverage will be comprehensive but emphasis will be on the last half of the course.

Reading: Finish reading Chapter 13.

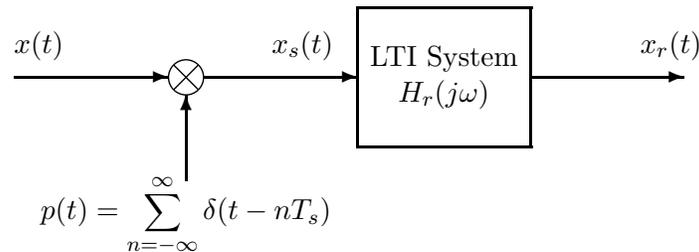
⇒ **Please check the “Bulletin Board” often. All official course announcements are posted there.**

The **STARRED** problems have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 12.1*:

Consider the following impulse train sampling and reconstruction system:

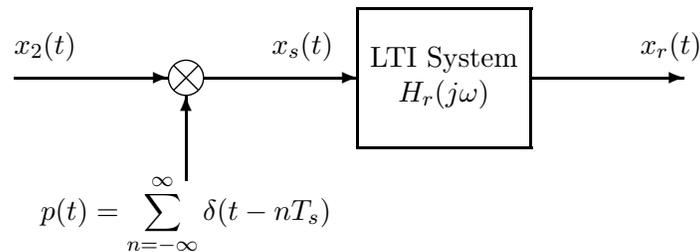


- If the input is $x(t) = 7 + 2 \cos(20\pi t) + 3 \cos(60\pi t + \pi/2)$, plot the Fourier transform $X(j\omega)$.
- Use the Sampling Theorem to determine the *maximum* value of T_s in the impulse train $p(t)$, so that there will be no aliasing when the input is $x(t)$ defined in part (a). In addition, determine the Nyquist rate for sampling this input signal.
- If $\omega_s = 2\pi/T_s = 100\pi$ rad/s and $x(t)$ is the signal defined in part (a), the condition of the Sampling Theorem is not satisfied and aliasing distortion occurs. Plot the Fourier transform $X_s(j\omega)$ for this case.
- Using the input signal from part (a) and the sampling rate of $\omega_s = 2\pi/T_s = 100\pi$ rad/sec., determine the Fourier transform $X_r(j\omega)$ of the output if the frequency response of the LTI system is

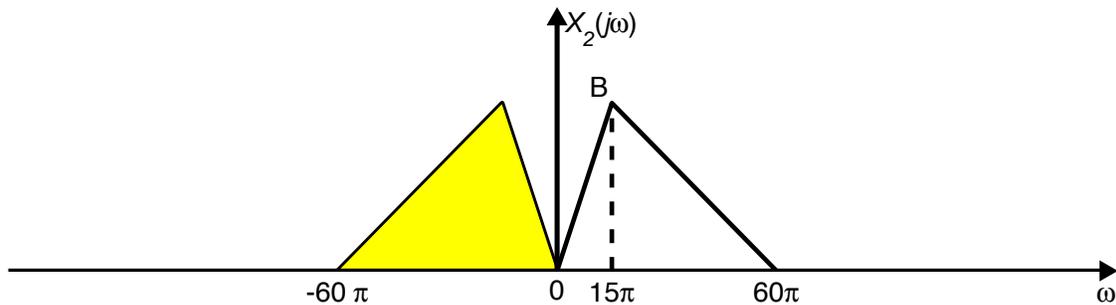
$$H_r(j\omega) = \begin{cases} 200 & |\omega| \leq \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$

PROBLEM 12.2*:

The Sampling Theorem involves the operations of impulse train sampling and reconstruction as shown in the following system:



The bandlimited Fourier transform of the input is depicted below (it is purely real):

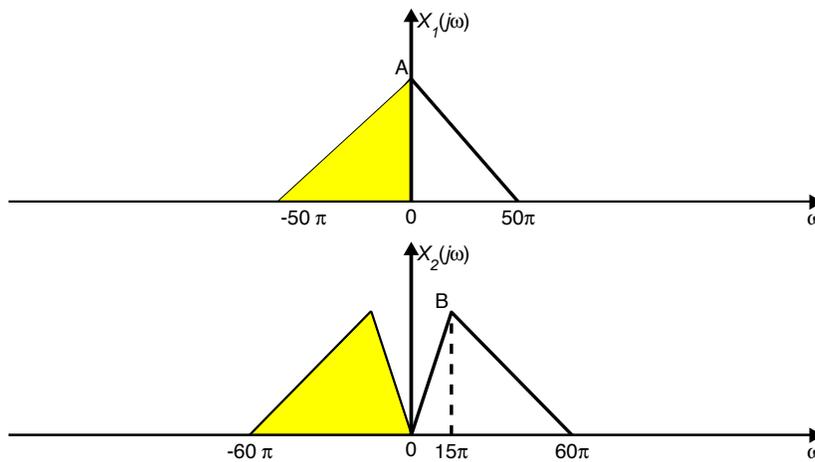
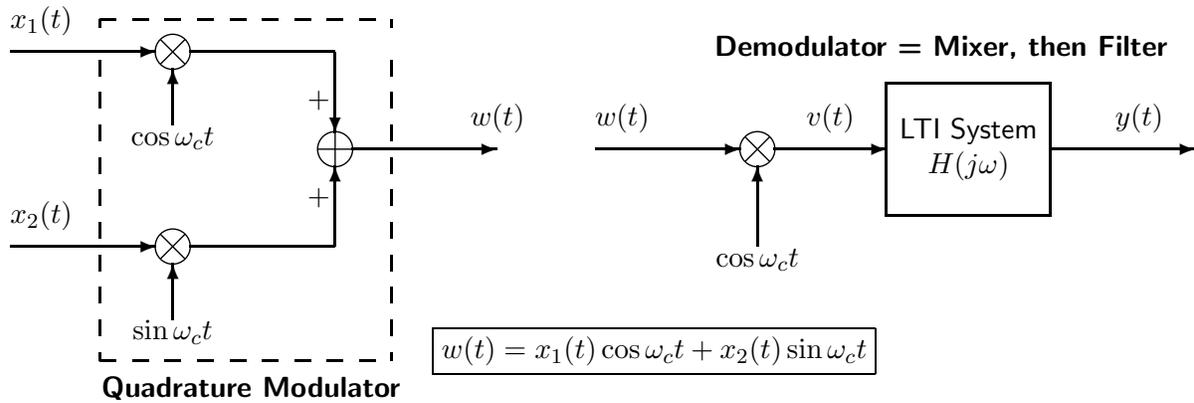


- For the input with Fourier transform depicted above, use the Sampling Theorem to choose the sampling rate $\omega_s = 2\pi/T_s$ so that $x_r(t) = x_2(t)$? Plot $X_s(j\omega)$ for the value of $\omega_s = 2\pi/T_s$ that is two times the Nyquist rate.
- If $\omega_s = 2\pi/T_s = 100\pi$ in the above system and $X_2(j\omega)$ is as depicted above, sketch the Fourier transform $X_s(j\omega)$ and show that aliasing occurs. There will be an infinite number of shifted copies of $X_2(j\omega)$, so indicate what the general pattern is versus ω .
- For the conditions of part (b), i.e., $\omega_s = 100\pi$, determine and sketch the Fourier transform of the output $X_r(j\omega)$ if the frequency response of the LTI system is

$$H_r(j\omega) = \begin{cases} 200 & |\omega| \leq \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$

PROBLEM 12.3*:

The system in the dashed box below is called a *quadrature modulation system*. It is a method of sending two bandlimited signals over the same channel.

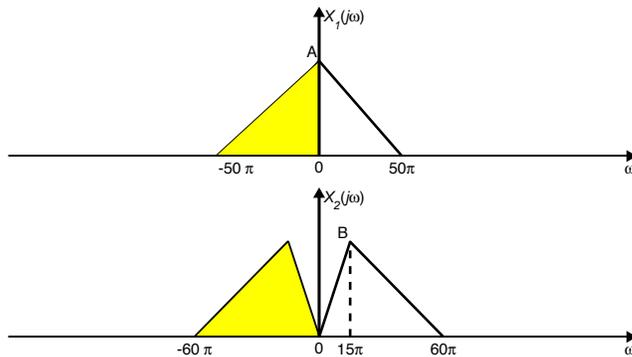
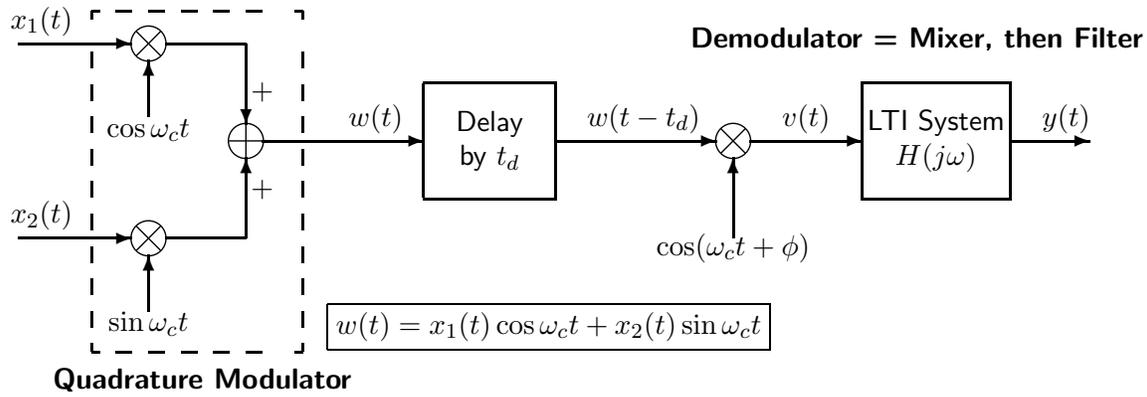


Assume that both input signals are bandlimited as shown above, i.e., $X_1(j\omega) = 0$ for $|\omega| \geq 50\pi$ and $X_2(j\omega) = 0$ for $|\omega| \geq 60\pi$ (and both Fourier transforms are purely real). Assume that the carrier frequency is $\omega_c = 500\pi$.

- Determine an expression for the Fourier transform $W(j\omega)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$. Make a sketch of $W(j\omega)$; you will have to sketch both the real and imaginary parts of $W(j\omega)$. Assume the simple shapes above for the bandlimited Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$, and use them in making your sketch of $W(j\omega)$.
- From the expression found in part (a) and the sketch that you drew, you should see that $W(j\omega) = 0$ for $|\omega| \leq \omega_a$ and for $|\omega| \geq \omega_b$. Determine ω_a and ω_b .
- The mixer is a device that multiplies by a sinusoid. Use the frequency shifting property of the Fourier transform to obtain the Fourier transform of $v(t)$. Make a sketch of $V(j\omega)$.
- The signal $v(t)$ as determined in part (c) is the input to an LTI system. Determine the frequency response of that system so that its output is $y(t) = x_1(t)$. Give your answer as a carefully labeled plot of $H(j\omega)$.

PROBLEM 12.4*:

The system in the dashed box below is called a *quadrature modulation system*. It is a method of sending two bandlimited signals over the same channel. In a realistic situation, the receiver might be located a long distance from the transmitter, so there is propagation delay between the transmitter and receiver which can be modeled as a perfect delay of t_d secs.



Use the same input signals and carrier frequency as the previous problem: Assume that both input signals are bandlimited as shown above, i.e., $X_1(j\omega) = 0$ for $|\omega| \geq 50\pi$ and $X_2(j\omega) = 0$ for $|\omega| \geq 60\pi$. Assume that the carrier frequency is $\omega_c = 500\pi$.

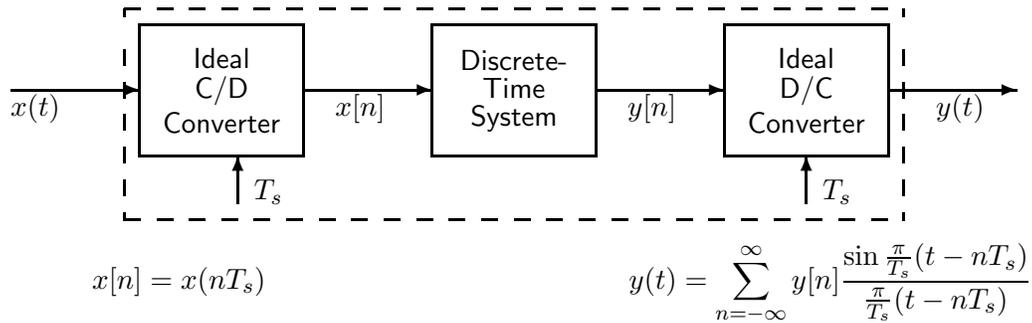
- The mixer is a device that multiplies by a sinusoid, but now we allow the phase of the mixer (ϕ) to be chosen. For this part, set $\phi = \frac{1}{2}\pi$, and assume that the channel delay is zero, $t_d = 0$. Use the frequency shifting property of the Fourier transform to obtain the Fourier transform of $v(t)$. Make a sketch of $V(j\omega)$ (real and imaginary parts).
- The signal $v(t)$ as determined in part (a) is then the input to the LTI system. If the frequency response of the $H(j\omega)$ system is an ideal LPF (i.e., $H(j\omega) = 1$ in the passband) with cutoff frequency of 100π , show that its output is $y(t) = Cx_2(t)$. Determine the constant C .
- For this part keep $\phi = \frac{1}{2}\pi$, but now assume that the delay is $t_d = 0.021$ secs. Use the same ideal LPF as in part(b).

Determine the output signal under these conditions. Explain your answer by deriving the Fourier transforms at both the input and output of the mixer. Give your answer as either formulas or sketches of these Fourier transforms (real and imaginary parts).

Comment: at the very beginning of the course, we discussed how the phase of a sinusoid was equivalent to time delay. Now we see a practical example of this relationship where the time delay of the transmission channel interacts with the phase of the mixer. If you dig deeper into the relationship you'll get basically the same equation that we derived for the sinusoid in Lecture #2.

PROBLEM 12.5*:

All parts of this problem are concerned with the following system.



In all parts of this problem, assume that the input signal $x(t)$ is bandlimited so that $X(j\omega) = 0$ for $|\omega| > 60\pi$ rad/s.

- Suppose that the discrete-time system is the *identity system* defined by $y[n] = x[n]$. What is the *minimum* value of the sampling frequency $\omega_s = 2\pi/T_s$ such that $y(t) = x(t)$?
- If the input is $x(t) = 7 + 2 \cos(20\pi t) + 3 \cos(60\pi t + \pi/2)$, the sampling frequency is $\omega_s = 100\pi$ rad/s, and $y[n] = x[n]$, plot $Y(j\omega)$ and give an equation for $y(t)$.
Hint: Minimize your work by using results from Problem 12.1.

- For the rest of the parts, assume that the input/output relation for the discrete-time system is

$$y[n] = x[n] + 2x[n - 1] + x[n - 2]$$

Make a plot of its frequency response $H(e^{j\hat{\omega}})$ versus $\hat{\omega}$ (discrete-time frequency).

- If the sampling frequency $\omega_s = 2\pi/T_s$ is large enough so that there is no aliasing of the input, and the filter is defined in part (c), the input and output Fourier transforms are related by an equation of the form $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$. If $\omega_s = 200\pi$ rad/s, find an equation for the overall effective frequency response $H_{\text{eff}}(j\omega)$, and plot the magnitude $|H_{\text{eff}}(j\omega)|$ and the phase $\angle[H_{\text{eff}}(j\omega)]$ versus analog frequency in rad/s.

****Note:** the *effective frequency response* $H_{\text{eff}}(j\omega)$ is only defined for $|\omega| < \frac{1}{2}\omega_s$ because the ideal D/C converter contains an ideal lowpass filter that is zero for $|\omega| > \frac{1}{2}\omega_s$. Consult the presentation in Lecture #22 for more details about the *effective frequency response*.