ECE 2025   Spring 2000
Problem Set #8

Assigned: 4-Mar-00
Due Date: Week of 20-Mar-00

There will be a lab quiz at the beginning of lab during the week of 20–23 March.

Any grading questions on Quiz #2 must be resolved no later than 22-March; after that date the scores will not be changed.

Reading: In *DSP First*, Chapter 7 on *The z-Transform.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 8.1*:

We now have four ways of describing an LTI system: the difference equation with filter coefficients $\{b_k\}$; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the system function $H(z)$.

(a) $y[n] = 3(x[n] - x[n-3])$.

(b) $h[n] = -\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3]$.

(c) $H(e^{j\hat{\omega}}) = [2j\sin(2\hat{\omega})]e^{-j3\hat{\omega}}$.

(d) $h[n] = \delta[n] + \delta[n-3]$.

PROBLEM 8.2*:

For each of the four systems given in Problem 8.1, determine the poles and zeros and make a plot of the pole-zero locations in the z-plane. Show the unit circle for reference.
Note: you can check your work with the MATLAB function `zplane()`, or the equivalent function `zzplane()` from the class web site.
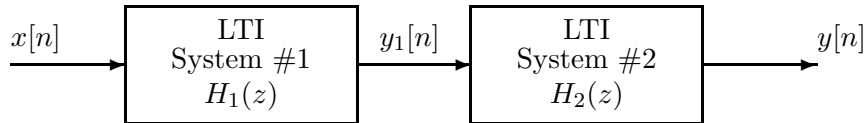
Figure 1: Cascade connection of two LTI systems.

PROBLEM 8.3*:

The diagram in Figure 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

In Figure 1, assume that both systems are first difference filters; i.e.,

$$y_1[n] = (x[n] - x[n-1]) \quad \text{and} \quad y[n] = (y_1[n] - y_1[n-1]).$$

(a) Determine the system function $H(z) = H_1(z)H_2(z)$ for the overall system.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, determine the impulse response $h[n]$ of the overall system in Figure 1.

(d) From $H(z)$, obtain an expression for the frequency response $H(e^{j\hat{\omega}})$ of the overall cascade system.

(e) Use your result from (d) as an aid in sketching the frequency response (magnitude and phase) functions of the overall cascade system for $-\pi \leq \hat{\omega} \leq \pi$.

PROBLEM 8.4*:

The system function of a linear time-invariant filter is given by the formula

$$H(z) = (1 + z^{-1})(1 - e^{j2\pi/3}z^{-1})(1 - e^{-j2\pi/3}z^{-1})$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. Make sure that all the filter coefficients $\{b_k\}$ in your difference equation are purely real.

(b) Use multiplication of z-transform polynomials to find the output when the input is

$$x[n] = -\delta[n-2] - \delta[n-3] - \delta[n-4].$$

(c) Plot the poles and zeros of $H(z)$ in the z-plane.

(d) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of frequency $\hat{\omega}$ will the output signal be zero for all n (i.e., $y[n] = 0$) ? Find all possible frequencies in the range $-\pi \leq \hat{\omega} \leq \pi$. *Hint: Take a look at the locations of the zeros of $H(z)$ as plotted in part (c).*

PROBLEM 8.5*:

The input to the C-to-D converter in the figure below is

$$x(t) = 10 + 10\cos(5000\pi t - \pi/2) + 10\cos(12000\pi t - \pi/5)$$

The system function for the LTI system is $H(z)$, and the sampling frequency of the A/D and D/A is $f_s = 10,000$ samples/second.
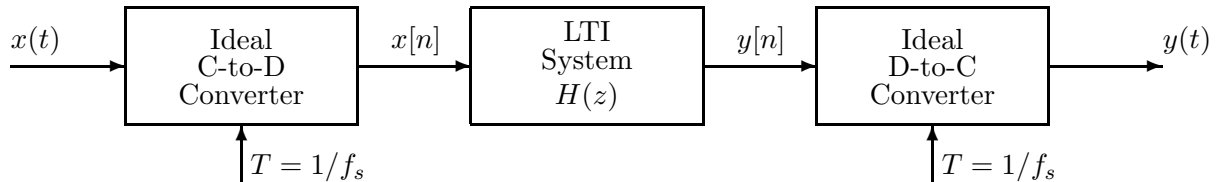
Suppose that the system above corresponds to the following MATLAB program:

```
nn = 0:16000;
fs = 10000;
tn = nn/fs;
xx = 10 + 10*cos(5000*pi*tn-pi/2) + 10*cos(12000*pi*tn-pi/5);
yy = conv([3,0,0,-3],xx);
soundsc(yy,fs)
```

(a) What is the system function $H(z)$ of the system that is implemented by the conv( ) statement?

(b) What is the frequency response of the system?

(c) Make a sketch of the spectrum of the discrete-time signal $x[n]$, the input to $H(z)$.

(d) Make a sketch of the spectrum of the discrete-time signal $y[n]$, the output from $H(z)$.

(e) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the soundsc( ) statement.