GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Lab #9: The z, n, and $\hat{\omega}$ Domains

Date: 2–8 Nov 1999

There will be a lab quiz at the beginning of the next lab: Lab #10.

NOTE: The lab verification consists of a number of questions that you will have to answer about the relationship between the time and frequency domains, as well as the poles and zeros.

This is *the official* Lab #9 description; it is derived from Lab C.10 in Appendix C of the text.

The lab report for this lab will be **INFORMAL:** discuss your filter design results from section 4. Staple the **Instructor Verification** sheet to the end of your lab report.

The report will **due during the week of 9–15 Nov. at the start of your lab**.

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1 Introduction & Objective

The objective for this lab is to build an intuitive understanding of the relationship between the location of poles and zeros in the z-domain, the impulse response $h[n]$ in the n-domain, and the frequency response $H(e^{j\hat{\omega}})$ (the $\hat{\omega}$-domain). A graphical user interface (GUI) called `pez` was written in MATLAB for doing interactive explorations of the three domains.[1]

2 Background

CD-ROM
PEZ

Invoke `pez` by simply typing `pez` at the MATLAB prompt, if you have the *DSP First Toolbox* installed. A control panel with a few buttons and a plot of the unit circle in the complex z-plane will pop up. You can use the control window to selectively place poles and zeros in the z-plane, and then observe (in a second window) how their placement affects the impulse and frequency responses. If the plots need manual updating, click on the `Redo Plots` button under the `<Quicksize...>` menu.

The `Real Time Drag Plots` button will put `pez` in a mode such that an individual pole/zero (pair) can be moved around and the corresponding $H(e^{j\hat{\omega}})$ and $h[n]$ plots will be updated as you drag the pole (or zero).

Since exact placement of poles and zeros with the mouse is difficult, an `Edit By Co-Ord` button is provided for numerical entry of the real and imaginary parts, or magnitude and angle (a separate edit window appears when you use this option). Before you can edit a pole or zero, however, you must first select it with the mouse. Removal of individual poles or zeros can also be performed by clicking on the `Delete Poles & Zeros` button (again, a separate window will appear). Note that all poles and/or zeros can be easily cleared by clicking on the `<Clear...>` menu, and then selecting `Poles`, `Zeros`, or `All`.

[1] `pez` was written by Craig Ulmer. It is part of the *DSP-First Toolbox*, so you can run it with MATLAB on your own computer.

3 Warm-up

Play around with `pez` for a few minutes to gain some familiarity with the interface. Implement the following first-order system:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$

by placing its poles and zeros at the correct location in the z-plane.

Instructor Verification (separate page)

3.1 Relationships between z, n, and $\hat{\omega}$ domains

CD-ROM

Instructor Verification Sheet

Work through the following warm-up exercises and keep track of your observations by filling in the worksheet at the end of this assignment. In general, you want to make note of the following quantities which vary as a function of the pole location(s):

- How does $h[n]$ change with respect to its rate of decay? For example, if $h[n] = a^n u[n]$, the impulse response will fall off more rapidly when a is smaller.
- If $h[n]$ exhibits an oscillation period, what is the period? Also, estimate the decay rate of the signal's "envelope."
- How does $H(e^{j\hat{\omega}})$ change with respect to peak location and peak width?

Note: review the "3-Domain Movies" in Chapters 7 and 8 on the *DSP-First* CD-ROM for examples of these relationships.

3.2 Real Poles

(a) Use `pez` to place a pole at $z = \frac{1}{2}$. You may have to use the `Edit by Co-Ord` button to get the location exactly right. Use the plots for this case as the reference for answering the next four parts.

(b) Move the pole close to the origin (still on the real axis). You can do this by clicking on the pole and dragging it to the new location. Describe the changes in the impulse response $h[n]$ and the frequency response $H(e^{j\hat{\omega}})$.

(c) You can also move poles and zeros under the influence of the `Real Time Drag Plots` option in `pez`. When this box is checked, the impulse response and frequency response plots are updated while you move the pole (or zero). Once this mode is set, click on the pole you want to move and start to drag it slowly. Watch for the update of the plots in the secondary window. After the real-time updating has started, you can release the mouse button and the pole (or zero) will follow the cursor. Click on the pole once more to stop moving it and to stop the updating. It is sometimes a little tricky to use. Also the display may be jerky unless you have a high-performance computer with fast graphics.

Move the real pole slowly from $z = \frac{1}{2}$ to $z = 1$ and observe the changes in the impulse response $h[n]$ and the frequency response $H(e^{j\hat{\omega}})$.

(d) Place the pole exactly on the unit circle. Describe the changes in $h[n]$ and $H(e^{j\hat{\omega}})$.

(e) Move the pole outside the unit circle. Describe the changes in $h[n]$. (Strictly speaking, $H(e^{j\hat{\omega}})$ does not exist in this case.)

(f) In general, where should poles be placed to guarantee system stability? By stability we mean that the system's output does not blow up.

3.3 Complex Poles and Zeros

If the denominator polynomial $A(z)$ has a complex root, it will have a second root at the conjugate location when the polynomial coefficients are real. For example, if we place a root at $z = \frac{1}{3} + j\frac{1}{2}$, then we will also get one at $z = \frac{1}{3} - j\frac{1}{2}$.

(a) What property of the polynomial coefficients of $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ will guarantee that the roots come in conjugate pairs?

(b) Clear all the poles and zeros from `pez`. Now place a pole with magnitude 0.75 at an angle of $45°$; and then two zeros at the origin ($z = 0$). Note that `pez` automatically places a conjugate pole in the z-domain. The frequency response has a peak—record the frequency (location) of this peak.

(c) Change the angle of the pole: move the pole to $90°$, then $135°$. Describe the changes in $|H(e^{j\hat{\omega}})|$. Concentrate on the location of the peak.

Next, we will put complex zeros on the unit circle to see the effect on $|H(e^{j\hat{\omega}})|$.

(d) Clear all poles and zeros from `pez`. Now place zeros at the following locations: $z_k = e^{j2\pi k/5}$, for $k = 1, 2, 3, 4$ (remember that conjugate pairs such as z_2 and z_3 will be entered simultaneously). In addition, put 4 poles at $z = 0$. Judging from the impulse response and frequency response what type of filter have you just implemented?

4 Laboratory: Filter Design

Filter design is a process that selects the coefficients $\{a_k\}$ and $\{b_k\}$ to accomplish a given task. In this section, we will use `pez` to understand how placing the poles and zeros of $H(z)$ can lead to an excellent design of a bandpass filter with a desirable frequency response specifications.

4.1 Filter Coefficients from Roots

If the denominator polynomial $A(z)$ has a complex root, it will have a second root at the conjugate location when the polynomial coefficients are real.

(a) Derive the filter coefficients for the denominator $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ and numerator $B(z) = 1 + b_1 z^{-1} + b_2 z^{-2}$ when the poles are:

$$p_1 = 0.8 e^{j\pi/4}, \quad p_2 = 0.8 e^{-j\pi/4},$$

and the zeros are: $z_1 = 1, \quad z_2 = -1$.
Use the following relationship:

$$H_1(z) = G\frac{B(z)}{A(z)} = G\frac{(1 - z_1 z^{-1})(1 - z_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})} \tag{1}$$

where z_1 and z_2 are the zeros, and p_1 and p_2 are the poles defined above.
(Remember that MATLAB can multiply polynomials via its `conv()` function. Also, `pez` has a display mode that will show the exact values of the filter coefficients as well as the poles and zeros.)

(b) Determine G so that the maximum magnitude of $H_1(e^{j\hat{\omega}})$ is one, and then make a plot of the frequency response.
(Remember that MATLAB can find the maximum value and its location with the `max()` function.)

(c) Determine the frequency location $(\hat{\omega})$ where the maximum magnitude of $H_1(e^{j\hat{\omega}})$ occurs—call this the *center frequency*.

(d) This filter is a bandpass filter, so we can measure its *passband width*. which is defined (here) are the distance between the 90% points on the frequency response. In order to do this, find the frequencies where $|H_1(e^{j\hat{\omega}})|$ is equal to 0.9; one is above the center frequency and one is below. Subtract these two frequencies to compute the bandwidth.
(Although the frequency response plot has two peaks, one is in negative frequency because we have symmetry so concentrate on measuring the bandwidth of the peak in positive frequency.)

4.2 Wider Bandpass Filter

The task here is to create a bandpass filter that has a wdier passband. In order to be precise about this we need a set of specifications:

1. *Normalization:* the maximum value of the frequency response (magnitude) should be one.
2. *Passbands:* for the region $0.3\pi \leq |\hat{\omega}| \leq 0.5\pi$, the magnitude response should be greater than 0.9. Notice that $|\hat{\omega}|$ defines two regions: one in positive frequency and one in negative frequency.
3. *Stopbands:* for the regions $|\hat{\omega}| \leq 0.01\pi$ and $0.75\pi \leq |\hat{\omega}| \leq \pi$, the magnitude response should be less than 0.1.
4. When converted to radians, the *cutoff frequencies* are: $0.1\pi = 0.3142, 0.3\pi = 0.9425, 0.5\pi = 1.5708$, and $0.75\pi = 2.3562$. These are also called the *band edges*.

This *wider bandpass* filter can be synthesized from the cascade of two second-order filters, which gives an overall system with 4 poles and 4 zeros—a fourth-order filter. One possible result shown in Fig. 1. In this lab you should use the second-order filter from Section 4.1 which we will call $H_1(z)$ and cascade another second-order filter which will be called $H_2(z)$. The design of $H_2(z)$ should be carried out as follows: Place the zeros of $H_2(z)$ at $z = +1$ and $z = -1$, but let the poles vary. The main point of this section of the lab is to use `pez` and some MATLAB code to find the pole locations for $H_2(z)$. (Note: the answer is not unique.)

(a) Start the process by using `pez` to locate all the known poles and zeros in $H_1(z)$ and $H_2(z)$. Then add the "unknown" pole pair needed in $H_2(z)$. Use `pez` to move the "unknown" pole pair around in the z-plane until you seem to get a reasonable magnitude response. In your lab report, describe how you did this and how you found a reasonable location for this "unknown" pole pair. Even though you canlocate the pole pair exactly with `pez`, this part is qualitative because the frequency response plot in `pez` is not detailed enough to see if you have met the specs.

(b) Now that you have used `pez` to get a good idea of where the "unknown" pole pair should be, write some MATLAB code to generate the precise frequency response of the overall (cascaded) system. In this code you will have two second-order filters.

You should use this MATLAB code to do a trial-and-error experiment of moving the "unknown" poles until you get a magnitude response that meets the filter specifications given above. You can use this MATLAB program to make a detailed plot of the frequency response to verify that you have met the specs.

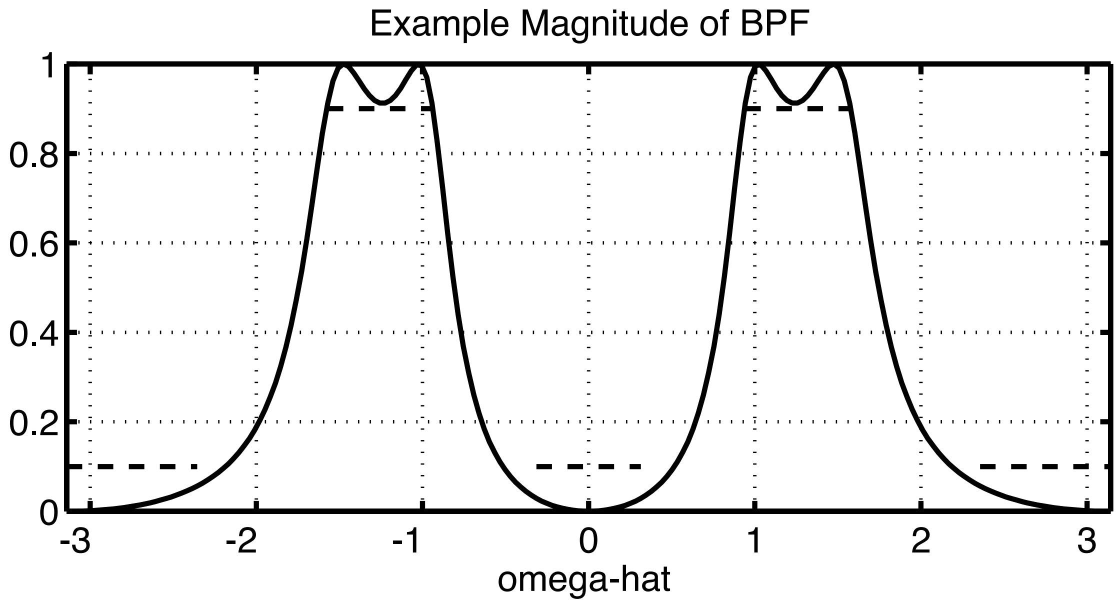

Figure 1: Magnitude response of a filter that meets the specs. The specs (at levels 0.9 and 0.1) are shown as dashed lines. Frequency axis is $\hat{\omega}$. You can use `pez` to help you find the pole locations that will produce a similar filter (but not the same one as this).

(c) One of the things needed in the previous part is a simple formula that links the pole locations and the filter coefficients for the denominator polynomial (the `aa` vector in MATLAB). Remember that MATLAB can do the conversion between root locations and polynomial coefficients: consult help on the MATLAB commands `roots` and `poly`.

You can also use the MATLAB function `zplane` (or the DSP-First function `zzplane`) to exhibit the pole-zero diagram of the overall system.

(d) You should also check your results by calculating the filter coefficients by hand (see the previous section on polynomials with complex coefficients). Record the coefficients of the IIR filter, $H_2(z)$.

(e) Write out by hand a single IIR difference equation that would have to be implemented for this *wider bandpass* filter. This is the processing equation that would take time-domain inputs $x[n]$ and produce outputs $y[n]$. It is a fourth-order system.

(f) Lastly you should make plots of the frequency responses of $H_1(z)$, $H_2(z)$ and the overall cascaded filter that you "designed" in part (b). Plot both the magnitude and phase of each filter. (Don't use `pez` for these plots because you cannot get a grid that shows the values of the frequency response.)

Then explain briefly how the frequency response magnitudes and phases combine to give the appropriate *wider bandpass* filter.

Lab #9
ECE-2025 Fall-1999
Instructor Verification

Name: ______________________________ Date of Lab: __________

Part 3: Implemented first-order system with `pez`. Explain the frequency response peak(s) and valley(s):

Verified:____________________ Date/Time:__________

Part	Observations
3.2(a)	$h[n]$ decays exponentially with no oscillations, $H(e^{j\hat{\omega}})$ has a hump at $\hat{\omega} = 0$
3.2(b)	
3.2(c)	
3.2(d)	
3.2(e)	
3.2(f)	
3.3(a)	
3.3(b)	
3.3(c)	
3.3(d)	

Verified:____________________ Date/Time:__________