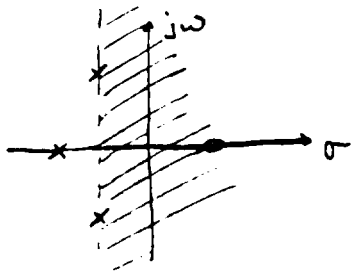
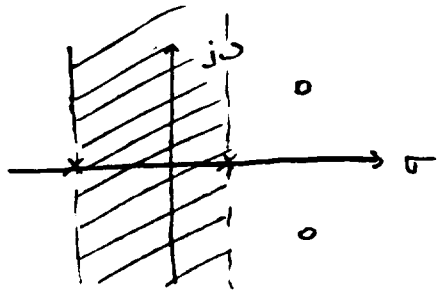
Solutions PS # 6 EE 3230 8/11/94

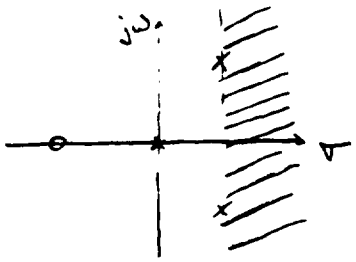
6.1) 5.16



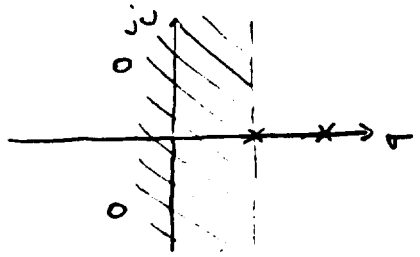
a



b



c



d

a) causal
b) not causal
c) marginally stable & causal
d) not causal

for the causal systems you would have to examine H(s) to be sure

(6.2) 5.19

$$H(s) = H_1(s)\,H_2(s) + \left[(H_4(s)\,H_5(s)) + H_3(s)\right] H_6(s)$$

b) $H_1(s) = \frac{1}{s}$, $H_2(s) = \frac{3}{s+3}$, $H_3(s) = H_5(s) = \frac{1}{s+2}$,

$H_4(s) = \frac{1}{s+1}$, $H_6(s) = 1 - \frac{2}{s+3} = \frac{s+1}{s+3}$

$$H_s = \frac{3}{s(s+3)} + \left[\left(\frac{1}{s+1}\right)\left(\frac{1}{s+2}\right) + \frac{1}{s+2}\right]\frac{s+1}{s+3}$$

$$= \frac{3}{s(s+3)} + \frac{1}{(s+2)(s+3)} + \frac{s+1}{(s+3)(s+2)}$$

$$= \frac{3s+6}{s(s+3)(s+2)} + \frac{s}{s(s+2)(s+3)} + \frac{s^2+s}{s(s+3)(s+2)} = \frac{s^2+5s+6}{s(s+3)(s+2)}$$

$$= \frac{(s+3)(s+2)}{s(s+3)(s+2)} = \frac{1}{s} \qquad \mathrm{Re}\{s\} > 0$$

6.3) 5.20

a) $W(s) = X(s) + Y(s)G(s)$

$Y(s) = W(s)F(s) \Rightarrow W(s) = \frac{Y(s)}{F(s)}$

$\frac{Y(s)}{F(s)} = X(s) + Y(s)G(s) \Rightarrow Y(s)\left[\frac{1}{F(s)} - G(s)\right] = X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{\frac{1}{F(s)} - G(s)} = \frac{1}{\frac{1 - G(s)F(s)}{F(s)}} = \frac{F(s)}{1 - G(s)F(s)}$$

b) i) $H(s) = \frac{1}{1 + s/\omega_b} = \frac{\omega_b}{\omega_b + s}$ LPF

ii) $H(s) = \frac{s/\omega_b}{1 + s/\omega_b} = \frac{s}{\omega_b + s}$ HPF

iii) $H(s) = \frac{-K}{1 + KG(s)} \approx -\frac{1}{G(s)}$

iv) $H(s) = \frac{F(s)}{1 + KF(s)} \approx \frac{1}{K}$

6.4) 5.22

a) $H(s) = 2 - \frac{1}{s} - \frac{3}{s+2} = \frac{2(s(s+2)) - s+2 - 3s}{s(s+2)}$

$= \frac{2s^2+4s-s-3s+2}{s(s+2)} = \frac{2(s^2+1)}{s(s+2)}$

$H_I(s) = \frac{s(s+2)}{2(s^2+1)}$ stable

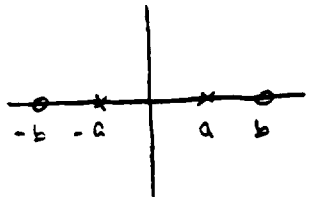
b) $H(s) = 1 - \left[\frac{1}{s+1} - \frac{1}{s-1}\right] = \frac{s^2+1}{s^2-1}$

$H_I(s) = \frac{s^2-1}{s^2+1}$ not stable cause poles on $j\omega$ axis

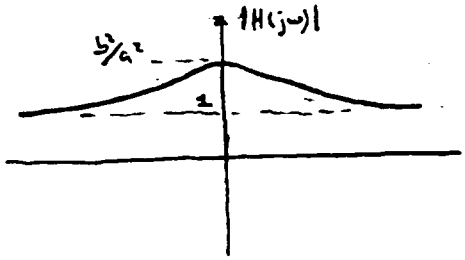
c) $H(s) = \frac{1}{s-1} - \frac{e^T e^{-sT}}{s-1} = \frac{1-e^T e^{-sT}}{s-1}$

$H_I(s) = \frac{s-1}{1-e^T e^{-sT}}$ stable

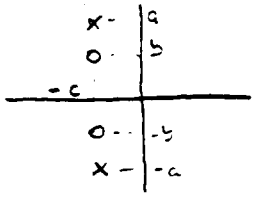
(6.5) 5.30



$$H(s) = \frac{s^2-b^2}{s^2-a^2} \Rightarrow H(j\omega) = \frac{\omega^2+b^2}{\omega^2+a^2}$$



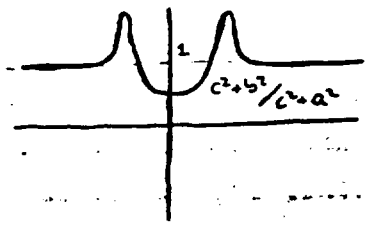
b)



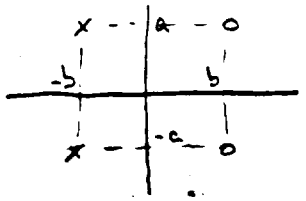
$$H(s) = \frac{[s+(c-jb)][s+(c+jb)]}{[s+(c-ja)][s+(c+ja)]}$$

$$= \frac{s^2+sc+sjb+sc-sjb+c^2+b^2}{s^2+sc+sja+sc-sja+c^2+a^2}$$

$$H(s) = \frac{s^2+2sc+c^2+b^2}{s^2+2sc+c^2+a^2} \Rightarrow H(j\omega) = \frac{-\omega^2+2j\omega c+c^2+b^2}{-\omega^2+2j\omega c+c^2+a^2}$$



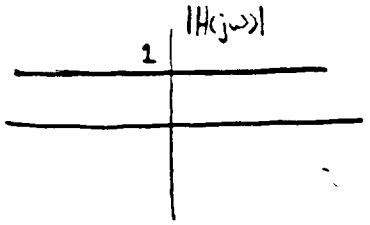
c)



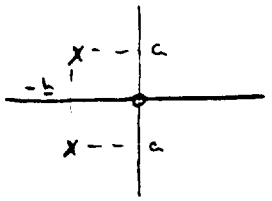
$$H(s) = \frac{[s-(b+ja)][s-(b-ja)]}{[s+(b-ja)][s+(b+ja)]}$$

$$= \frac{s^2 - sb + sja - sb - sja + b^2 + a^2}{s^2 + sb + sja + sb - sja + b^2 + a^2}$$

$$H(s) = \frac{s^2 - 2sb + b^2 - c^2}{s^2 + 2sb + b^2 - a^2} \Rightarrow H(j\omega) = \frac{-\omega^2 - 2j\omega b + b^2 + a^2}{-\omega^2 + 2j\omega b - b^2 + a^2}$$



d)



$$H(s) = \frac{s}{[s+(b-ja)][s+(b+ja)]} = \frac{s}{s^2 + sb + sja + sb - sja + b^2 + a^2}$$

$$H(s) = \frac{s}{s^2 + 2sb + b^2 + a^2}$$

$$H(j\omega) = \frac{j\omega}{-\omega^2 + 2j\omega b + b^2 + a^2}$$