RESERVE DESK

EE 3230 Solns PS # 4

SEP 27 1995 7/29/94

4.1) 4.14

i) $X(j\omega)$ real $\Rightarrow$ $X(j\omega) = X^*(j\omega)$

since $X(j\omega) \to x(t)$ and $X^*(j\omega) \to x^*(-t)$

$x(t) = x^*(-t)$ i.e. conjugate symm.

since $x(t)$ real $\Rightarrow$ $x(t)$ is even

c, f, h

ii) $X(j\omega)$ imag $\Rightarrow$ $X(j\omega) = -X^*(j\omega)$

since $X(j\omega) \to x(t)$ and $-X^*(j\omega) \to -x^*(-t)$

$x(t) = -x^*(-t)$ ie conjugate antisymm

since $x(t)$ real $\Rightarrow$ $x(t)$ is odd

b, e, g

iii) $X(0) = 0 \Rightarrow \int_{-\infty}^{\infty} x(t)\,e^{-j(0)t} = 0 \Rightarrow \int_{-\infty}^{\infty} x(t) = 0$

a, b, d, e, f, g, h

iv) $\int_{-\infty}^{\infty} X(j\omega)\,d\omega = 0 \Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,e^{+j\omega(0)}\,d\omega = x(t=0) = 0$

b, c, d, e, g

v) $X(j\omega)$ periodic implies $x(t)$ consists of impulses only. This is because we know that a periodic signal in time can be expressed as a sum of impulses whose values are the Fourier series coefficients. ~~So the spectrum~~ would be impulses. Apply duality to get our case.

c, d

vi) $e^{j\omega\tau} X(j\omega) = [e^{j\omega\tau} X(j\omega)]^* = e^{-j\omega\tau} X^*(j\omega)$

$$e^{j\omega\tau} X(j\omega) \rightarrow x(t+\tau) \quad \text{and} \quad e^{-j\omega\tau} X^*(j\omega) \rightarrow x^*(-t-\tau)$$

so $x(t+\tau) = x^*(-t-\tau)$ for some $\tau \neq 0$

since $x(t)$ is real

a, d, F, g, h

$x(t+\tau)$ is ~~odd~~

vii) $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = E \implies \int_{-\infty}^{\infty} |x(t)|^2 \, dt = E$

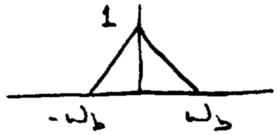
by Parseval

a, b, c

viii) $\int_{-\infty}^{\infty} \omega X(j\omega)\, d\omega = 0 \implies \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega\, X(j\omega) e^{j\omega(0)} d\omega \implies \left.\frac{d\,x(t)}{dt}\right|_{t=0} = 0$

b, c, d, e, F

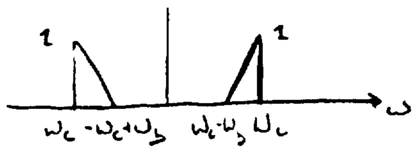
4.2) 4.18 assume $X(j\omega) =$



$V(j\omega) =$

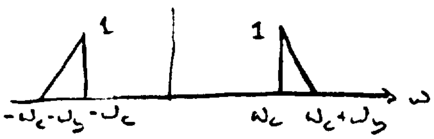
$W(j\omega) =$

$V(j\omega) + W(j\omega) =$



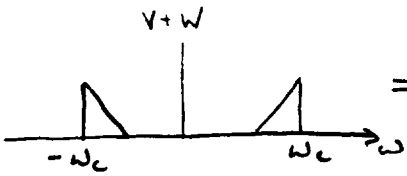
$= X_-(j\omega)e^{j\omega_c} + X_+(j\omega)e^{-j\omega_c}$

$V(j\omega) - W(j\omega) =$

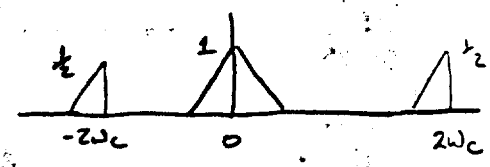


$= X_+(j\omega)e^{j\omega_c} + X_-(j\omega)e^{-j\omega_c}$

b)



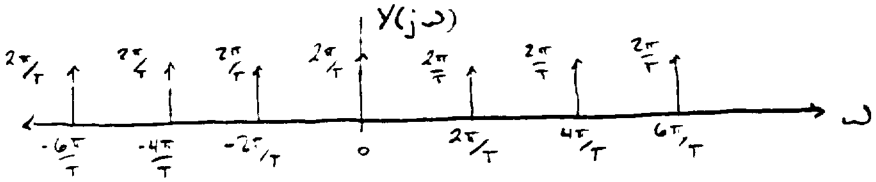
$\Rightarrow$ demodulation shifts each sideband by $+\omega_c$ and $-\omega_c$ yielding:



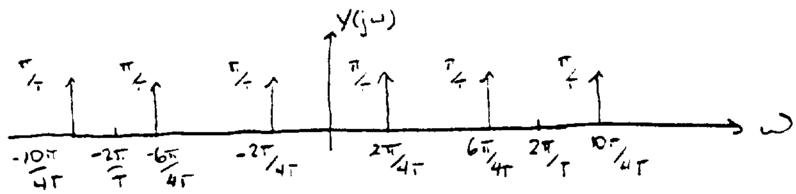
by low pass filtering, the component of interest (the one centered around $\omega=0$) can be removed

4.3) 4.23 $\quad Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$

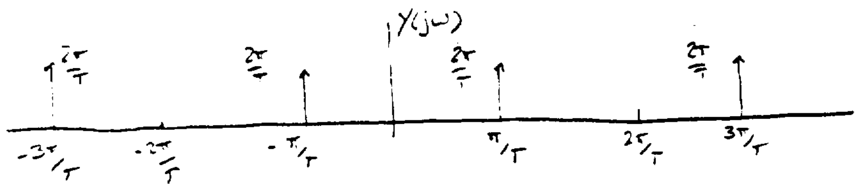
a) i) $x(t) = 1 \rightarrow X(j\omega) = 2\pi\delta(\omega)$.



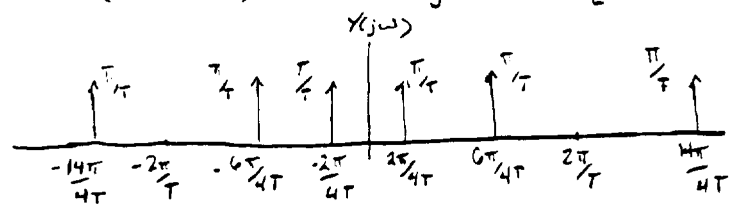
ii) $x(t) = \cos(\omega_s t/4) \rightarrow X(j\omega) = 2\pi\left[\frac{1}{2}\delta(\omega - \pi/2T) + \frac{1}{2}\delta(\omega + \pi/2T)\right]$



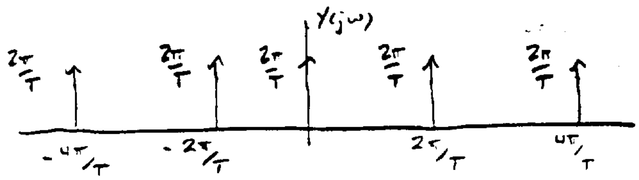
iii) $x(t) = \cos(\omega_s t/2) \rightarrow X(j\omega) = \left[\frac{1}{2}\delta(\omega - \pi/T) + \frac{1}{2}\delta(\omega + \pi/T)\right]2\pi$



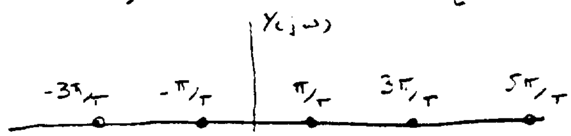
iv) $x(t) = \cos(3\omega_s t/4) \rightarrow X(j\omega) = 2\pi\left[\frac{1}{2}\delta(\omega - 6\pi/4T) + \frac{1}{2}\delta(\omega + 6\pi/4T)\right]$



v) $x(t) = \cos(\omega_s t) \rightarrow X(j\omega) = \left[\frac{1}{2}\delta(\omega - 2\pi/T) + \frac{1}{2}\delta(\omega + 2\pi/T)\right]2\pi$

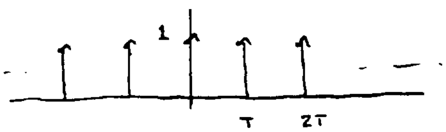


vi) $x(t) = \sin(\omega_s/2) \rightarrow X(j\omega) = \left[\frac{1}{2}\delta(\omega - \pi/T) - \frac{1}{2}\delta(\omega + \pi/T)\right]2\pi$

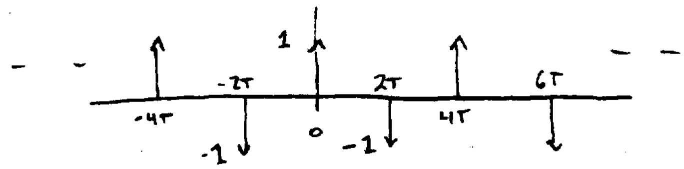


b) Cases iii) – vi)

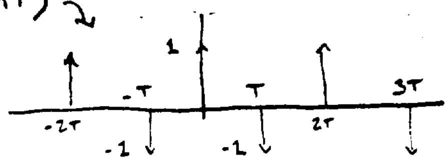
c) i)



ii) $y(t) = \sum_{n=-\infty}^{\infty} \cos\left(\frac{\pi}{2T} nT\right) \delta(t - nT)$



iii) $\sum_{n=-\infty}^{\infty} \cos\left(\pi/_T\, nT\right) \delta(t - nT)$ ↘



(see below)

4.4) 4.31

a) $H_{HP}(j\omega) = 1 - H_{LP}(j\omega) \Rightarrow h_{HP}(t) = \delta(t) - h_{LP}(t)$

b) $H_{BP}(j\omega) = H_{LP2}(j\omega) - H_{LP1}(j\omega) \Rightarrow h_{BP}(t) = h_{LP2}(t) - h_{LP1}(t)$

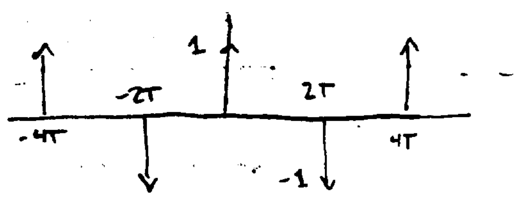
assumes $\omega_{b2} > \omega_{b1}$

c) $H_{BS}(j\omega) = 1 - H_{BP}(j\omega) \Rightarrow h_{BS}(t) = \delta(t) - h_{BP}(t)$

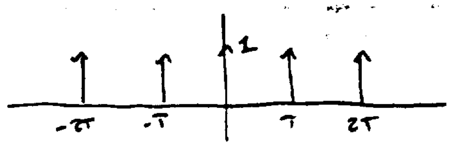
d) since each of the impulse responses integrate to a finite number they are all stable

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty \quad \forall \text{ the } h\text{'s in a–c}$$

4.23) c) cont iv) $\sum_{n=-\infty}^{\infty} \cos\left(3\pi/_{2T}\, nT\right) \delta(t - nT)$



v) $\sum_{n=-\infty}^{\infty} \cos\left(\frac{2\pi}{T} nT\right) \delta(t - nT)$



vi) $y(t) = 0$