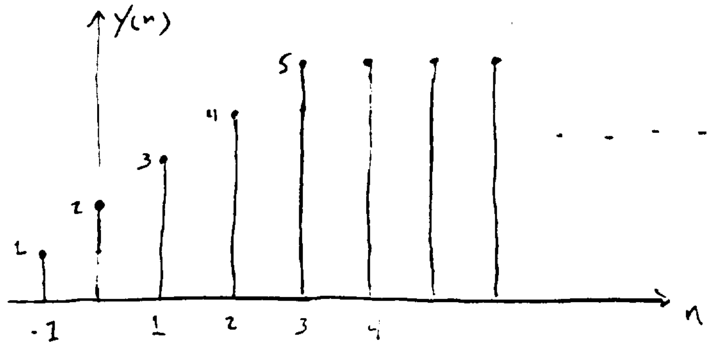
Solutions for PS # 3 EE 3230 7/18/94



3.1) problem 3.11

a) $x(n) = \sum_{k=-2}^{2} \delta(n-k)$

$$\Rightarrow y(n) = \sum_{k=-2}^{2} h(n-k) = u(n-1+2) + u(n-1+1) + u(n-1) + u(n-1-1) + u(n-1-2)$$

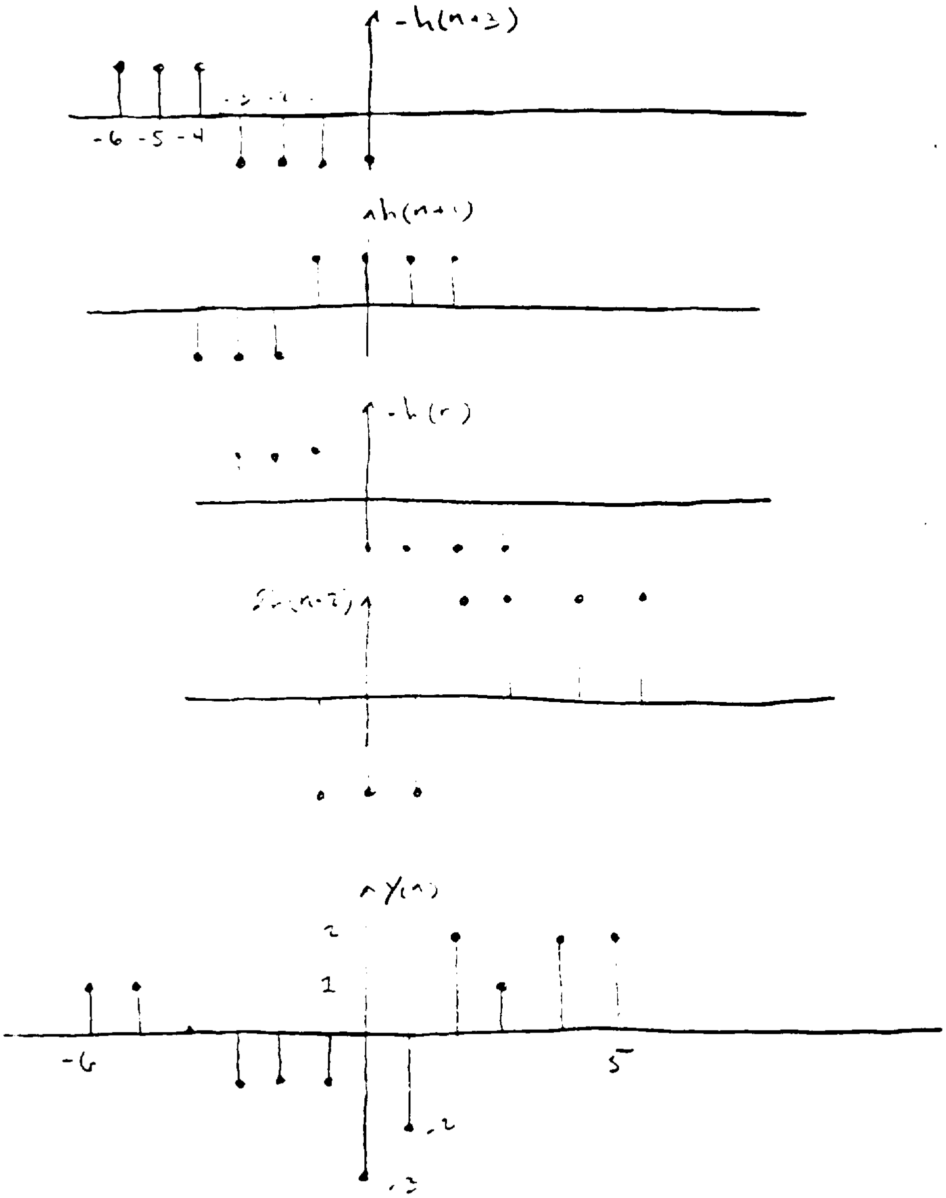



c) $x(n) = \delta(n) - a\delta(n-1)$

$$\Rightarrow y(n) = \delta(n) * a^n u(n) - a\delta(n-1) * a^n u(n)$$

$$= a^n u(n) - a \cdot a^{n-1} u(n-1) = a^n [\delta(n)]$$

c) $y(n) = -h(n+3) + h(n+1) - h(n) + 2h(n-2)$



3.2) problem 2.14

a) since $y(t) = \int_t^{\infty} x(\tau)d\tau \Rightarrow$ let $x(t) = \delta(t) \Rightarrow \int_t^{\infty} \delta(\tau)\,d\tau$

$= 1$ for $t < 0$, and $= 0$ for $t > 0 \Rightarrow h(t) = u(-t)$

not stable or causal

b) $y(t) = \int_0^{\infty} e^{\tau} x(t-\tau-1)\,d\tau \Rightarrow$ let $x(t) = \delta(t) \Rightarrow \int_0^{\infty} e^{\tau}\delta(t-\tau-1)\,d\tau$

$\tau$ needs to equal $t-1 \Rightarrow$ so if $t-1 > 0$ $h(t) = e^{t-1}$
if $t-1 < 0$ $h(t) = 0$

$\Rightarrow h(t) = e^{t-1}u(t-1)$ causal, not stable

3) problem 3.20

a) $x(t) \to \boxed{h(t)} \xrightarrow{y(t)} \boxed{d/dt} \to y'(t) \;=\; x(t) \to \boxed{d/dt} \xrightarrow{x'(t)} \boxed{h(t)} \to y'(t)$

$x(t) \to \boxed{h'(t)} \to y'(t)$ or $x'(t) \to \boxed{h(t)} \to y'(t)$

b) $x(t) \to \boxed{d/dt} \to \boxed{h(t)} \to \boxed{\int_{-\infty}^{t}} \to y(t) \;=\; x'(t) \to \boxed{h(t)} \to \boxed{\int_{-\infty}^{t}} \to y(t)$

or $x(t) \to \boxed{\int_{-\infty}^{t}} \to \boxed{h(t)} \to \boxed{d/dt} \to y(t) \;=\; x(t) \to \boxed{\int_{-\infty}^{t}} \to \boxed{h'(t)} \to y(t)$

either way yields $x(t) \to \boxed{h(t)} \to y(t)$

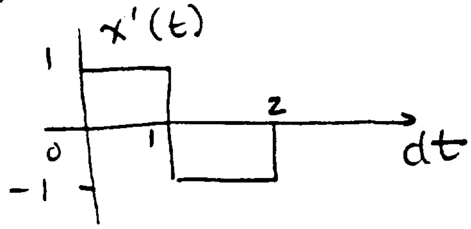
c) $x(t) \to \boxed{\int_{-\infty}^{t}} \to \boxed{h'(t)} \to y(t) \;=\; x(t) \to \cancel{\boxed{\int_{-\infty}^{t}}} \to \cancel{\boxed{d/dt}} \to \boxed{h(t)} \to y(t)$

$x(t) \to \boxed{h(t)} \to y(t)$

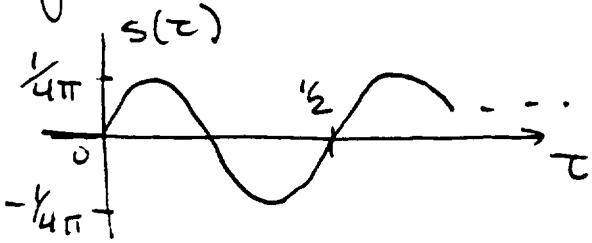
d) $x(t) \to \boxed{d/dt} \to \boxed{s(t)} \to y(t) \;=\; x(t) \to \cancel{\boxed{d/dt}} \to \cancel{\boxed{\int_{-\infty}^{t}}} \to \boxed{h(t)} \to y(t)$

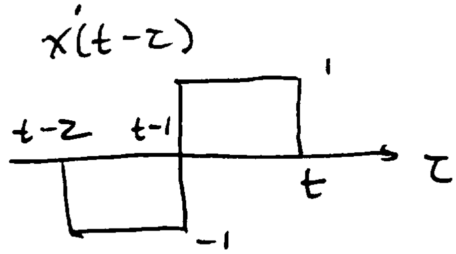
$x(t) \to \boxed{h(t)}$

(e) $s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau = u(t)\cdot\int_0^t \cos 4\pi\tau\,d\tau = \frac{1}{4\pi}\sin(4\pi t)\,u(t)$



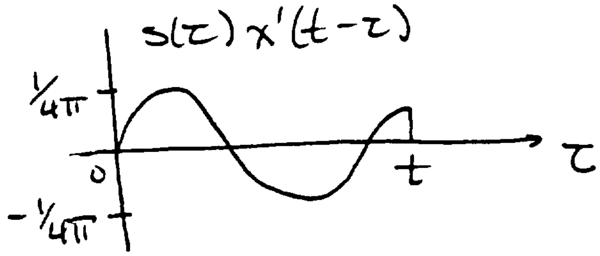

$y(t) = s(t) * x'(t)$




For $t<0$: $y(t) = 0$

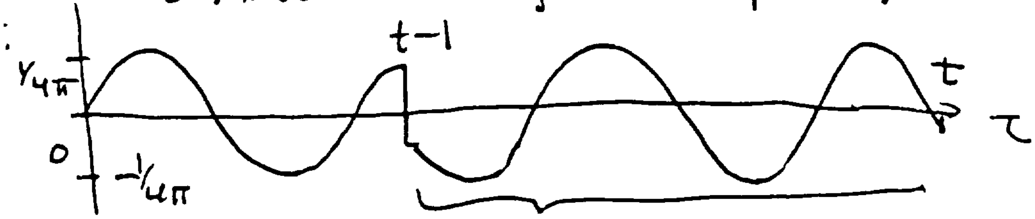
For $0<t<1$:




$$y(t) = \int_0^t \frac{1}{4\pi}\sin(4\pi\tau)\,d\tau = \cos 4\pi\tau\Big|_0^t = \cos 4\pi t$$

$-s(\tau)x'(t-\tau)$ (sorry about the sign error)

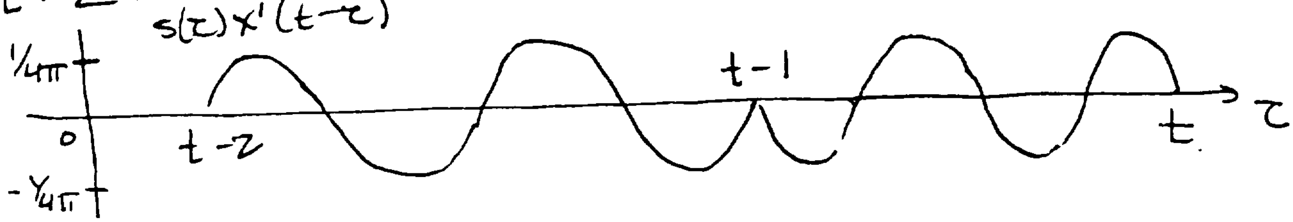
For $1<t<2$:




exactly 2 periods
⇒ integral is 0.

$$y(t) = -\int_0^t \frac{1}{4\pi}\sin(4\pi\tau)\,d\tau + 0 = -\cos 4\pi t$$

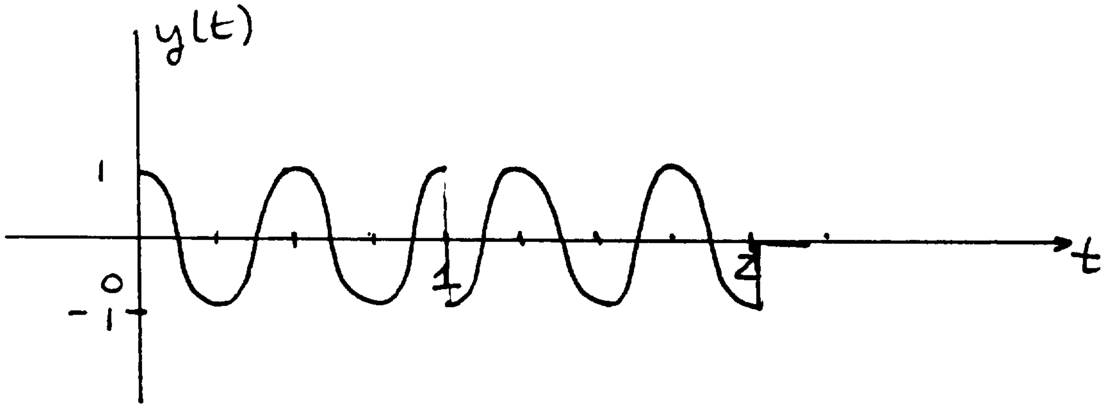
For $t>2$:



⇒ Always get 2 integrals that each cover exactly 2 periods. So integrals are <u>always</u> 0.

$$\Rightarrow y(t) = 0 .$$

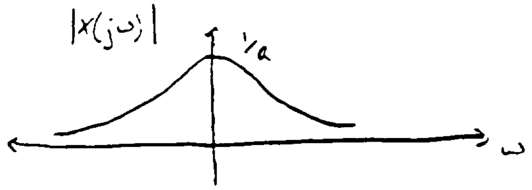
$$y(t) = \begin{cases} 0 , & t < 0 \\ \cos 4\pi t , & 0 < t < 1 \\ -\cos 4\pi t , & 1 < t < 2 \\ 0 , & t > 2 \end{cases}$$



3.4) problem 4.8

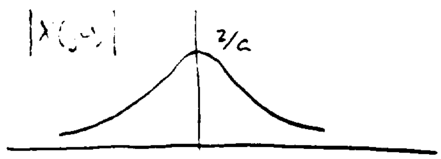
a) $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} = \int_{-\infty}^{0} e^{at}e^{-j\omega t} = \left.\frac{e^{(a-j\omega)t}}{a-j\omega}\right|_{-\infty}^{0} = \frac{1}{a-j\omega}$

$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$




b) $X(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\,dt + \int_{0}^{\infty} e^{-(a+j\omega)t}$

$$\frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$




c) $X(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega t}\delta(t-t_0) + \int_{-\infty}^{\infty} e^{-j\omega t}\delta(t+t_0)$

$$e^{-j\omega t_0} + e^{j\omega t_0} = 2\cos(\omega t_0)$$