GEORGIA INSTITUTE OF TECHNOLOGY

School of Electrical and Computer Engineering

Final Exam

Date: June 13, 1997 Course: EE 3230

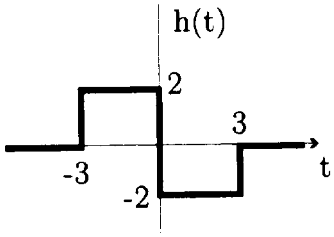
Name: ______________________________
Last, First

- Closed book, closed notes, and no calculators are allowed. Three $8\frac{1}{2}'' \times 11''$ handwritten sheets and the tables from the previous quizzes are allowed. Two hour and fifty minute time limit.
- None of the problems require involved calculations. Reconsider your approach before doing something tedious.
- All work should be performed on the quiz itself. If more space is needed, use the backs of the pages.

Problem	*Score*
1	
2	
3	
4	
5	
6	
Total	

Problem 1:

A linear, time-invariant system has the impulse impulse:



(a) Sketch $h(1-3t)$.

(b) Simplify $\int_0^3 h(t)\delta(t-1)dt$.

(c) Find and sketch the system's step response $s(t)$.

Problem 2:

The output signal, $y(t)$, of a linear, time-invariant system is related to its input signal, $x(t)$ by

$$y(t) = \int_{-1}^{1} \tau x(t-\tau) d\tau$$

(a) What is $h(t)$, the impulse response of this system?

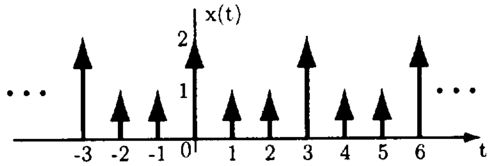
(b) Is the system stable? Why or why not?

(c) Is the system causal? Why or why not?

(d) What is $H(j\omega)$, the Fourier transform of $h(t)$?

Problem 3:

Consider the following periodic signal:



(a) Find $X(j\omega)$, the Fourier transform of $x(t)$.

(b) Find the Fourier series representation of $x(t)$.

(c) If $x(t)$ is passed through an ideal lowpass filter with cutoff frequency $\omega_c = \pi$ rad./s., what is the resulting output signal $y(t)$? Simplify your answer as much as possible.

Problem 4:

We know that $x(t) = e^{-at}u(t)$ has the Fourier transform $X(j\omega) = \frac{1}{a+j\omega}$ for $|a| > 0$. Using the properties of the Fourier transform, match the following signals with their transforms.

Signals	**Transforms**
$e^{-at}u(t+t_o)$	$\dfrac{1}{a+j\omega-j\omega_o}$
$Od\{e^{-at}u(t)\}$	$\dfrac{e^{-at_o}e^{-j\omega t_o}}{a+j\omega}$
$r(t) = u(t) * e^{-at}u(t)$	$\dfrac{e^{at_o}e^{j\omega t_o}}{a+j\omega}$
$e^{j\omega_o t}e^{-at}u(t)$	$\dfrac{ja}{(a+j\omega)^3}$
$t^2 e^{-at}u(t)$	$\dfrac{1}{ja\omega-\omega^2} + \frac{\pi}{a}\delta(\omega)$
	$\dfrac{-j\omega}{a^2+\omega^2}$
	$\dfrac{2}{(a+j\omega)^3}$
	None of the above

Problem 5:

A continuous-time system is given by the input/output differential equation:

$$\frac{d^2y(t)}{dt^2} + 3y(t) = 2x(t)$$

Note that this is equivalent to a simple pendulum model with input $x(t)$ being the driving force applied to the pendulum and output $y(t)$ equal to the angle of the pendulum relative to vertical.

(a) Compute the transfer function $H(s)$ of the system assuming zero initial conditions.

(b) Compute the impulse response $h(t)$ assuming zero initial condiations.

(c) Suppose that $y(0^-) = 0$, $\frac{dy(0^-)}{dt} = 1$, and the input force is $x(t) = A\delta(t)$. Determine the value of A such that the resulting response $y(t)$ is zero for *all* $t > 0$. In other words, we want the impulsive input force $A\delta(t)$ to "cancel" the nonzero initial velocity $\frac{dy(0^-)}{dt}$.

Problem 6:

A causal linear time-invariant discrete-time system is described by the transfer function

$$H(z) = \frac{3}{z^2 - \frac{1}{4}z - \frac{1}{8}}$$

(a) Find $h[n]$, the impulse response for this system.

(b) Does this impulse response have a discrete-time Fourier transform $H(e^{j\Omega})$? Why or why not?

(c) How many different time domain signals have this same $H(z)$ as their z-transform but with different regions of convergence?

TABLE 6.1 *Properties of the z Transform*

Property	*Time domain*	*Transform*	*ROC*
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$R' \supset R_1 \cap R_2$
Time shift	$x[n - n_0]$	$z^{-n_0}X(z)$	$R' \supset R \cap$ $0 < \|z\| < \infty$
Modulation	$z_0^n x[n]$	$X(z/z_0)$	$R' = \|z_0\| R$
	$e^{j\Omega_0 n}x[n]$	$X(ze^{-j\Omega_0})$	$R' = R$
Time reversal	$x[-n]$	$X(1/z)$	$R' = 1/R$
Differentiation	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	$R' = R$
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$R' \supset R_1 \cap R_2$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$X(z)\dfrac{1}{1 - z^{-1}}$	$R' \supset R \cap \|z\| > 1$

TABLE 6.2 *Common z Transforms*

Signal	*Time domain*	*Transform*	*ROC*
Impulse	$\delta[n]$	1	All z
	$\delta[n - n_0],\quad n_0 > 0$	z^{-n_0}	$\|z\| > 0$
	$\delta[n + n_0],\quad n_0 > 0$	z^{n_0}	$\|z\| < \infty$
Unit step	$u[n]$	$\dfrac{1}{1 - z^{-1}}$	$\|z\| > 1$
	$-u[-n - 1]$	$\dfrac{1}{1 - z^{-1}}$	$\|z\| < 1$
Exponential	$a^n u[n]$	$\dfrac{1}{1 - az^{-1}}$	$\|z\| > \|a\|$
	$-a^n u[-n - 1]$	$\dfrac{1}{1 - az^{-1}}$	$\|z\| < \|a\|$
Weighted exponential	$(n + 1)a^n u[n]$	$\dfrac{1}{(1 - az^{-1})^2}$	$\|z\| > \|a\|$
Causal sine	$(\sin \Omega_0 n)u[n]$	$\dfrac{(\sin \Omega_0)z^{-1}}{1 - 2(\cos \Omega_0)z^{-1} + z^{-2}}$	$\|z\| > 1$
Causal cosine	$(\cos \Omega_0 n)u[n]$	$\dfrac{1 - (\cos \Omega_0)z^{-1}}{1 - 2(\cos \Omega_0)z^{-1} + z^{-2}}$	$\|z\| > 1$
Damped sine	$r^n(\sin \Omega_0 n)u[n]$	$\dfrac{r(\sin \Omega_0)z^{-1}}{1 - 2r(\cos \Omega_0)z^{-1} + r^2z^{-2}}$	$\|z\| > r$
Damped cosine	$r^n(\cos \Omega_0 n)u[n]$	$\dfrac{1 - r(\cos \Omega_0)z^{-1}}{1 - 2r(\cos \Omega_0)z^{-1} + r^2z^{-2}}$	$\|z\| > r$