GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

Final Exam

Date: December 4, 1995 Course: EE 3230

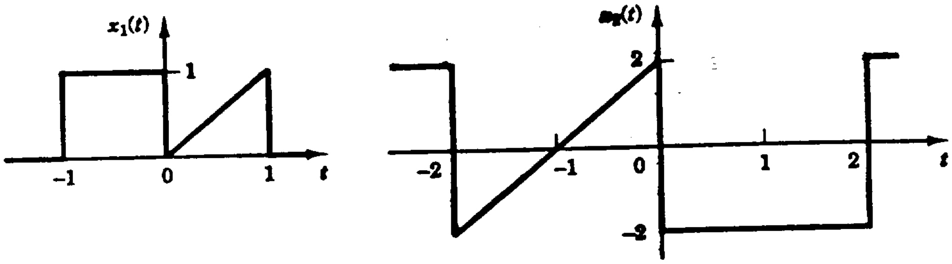
Name: ______________________________
Last, First

- Closed book, closed notes, and no calculators are allowed. Three $8\frac{1}{2}'' \times 11''$ handwritten sheets and the tables from the previous quizzes are allowed. Two hour and fifty minute time limit.
- None of the problems require involved calculations. Reconsider your approach before doing something tedious.
- All work should be performed on the quiz itself. If more space is needed, use the backs of the pages.

Problem	*Score*
1	
2	
3	
4	
5	
6	
7	
Total	

Problem 1:

Consider these two signals:



(a) Sketch $1 - x_1(1-t)$.

(b) Sketch $x_2(2t+1)$.

(c) Express $x_2(t)$ as a function of $x_1(t)$.

Problem 2:

Determine whether or not each of the following systems is memoryless, causal, stable, time-invariant, and/or linear.

(a) $y(t) = |x(t)|$

(b) $y(t) = \begin{cases} x(t) & , \text{if } x(t) \geq 0 \\ 0 & , \text{if } x(t) < 0 \end{cases}$

(c) $y(t) = \begin{cases} -10 & , \text{if } x(t) < 1 \\ 10x(t) & , \text{if } |x(t)| \leq 1 \\ 10 & , \text{if } x(t) > 1 \end{cases}$

Problem 3:

Given that $x(t) = e^{-|t|}$ has the Fourier transform $X(j\omega) = \frac{2}{\omega^2+1}$, find the Fourier transforms of:

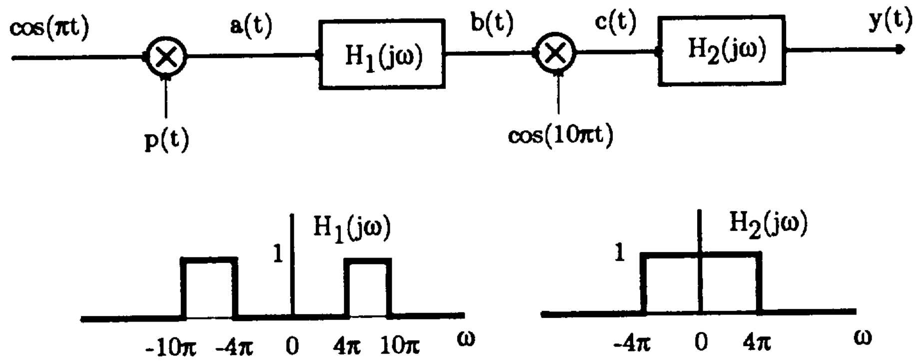
(a) $a(t) = \frac{d}{dt}e^{-|t|}$.

(b) $b(t) = \frac{1}{t^2+1}$.

(c) $c(t) = \frac{\cos(t)}{t^2+1}$.

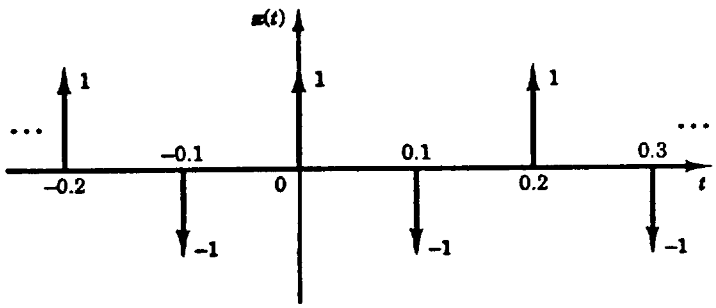
Problem 4:

For the system shown below with $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - 0.2n)$, accurately sketch $A(j\omega)$, $B(j\omega)$, $C(j\omega)$, and $Y(j\omega)$.



Problem 5:

Consider the periodic signal:



(a) The Fourier transform of $x(t)$ has the form $X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_o)$. Find ω_o and a_k.

(b) Give the Fourier series representation of $x(t)$.

(c) The signal $x(t)$ is input to a system with frequency response

$$H(j\omega) = \begin{cases} 1, & -7\pi < \omega < 12\pi \\ 0, & \text{otherwise} \end{cases}$$

Find the system output $y(t)$. Simplify your answer as much as possible.

Problem 6:

Consider the causal, LTI system with input $x(t)$ and output $y(t)$ described by

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = 2x(t)$$

(a) Find the transfer function of this system, i.e., the Laplace transform of its impulse response including the region of convergence.

(b) Find the impulse response of this system.

(c) Find the transfer function for this system's inverse system.

Problem 7:

Consider a causal, LTI system whose impulse response $h[n]$ has the z-transform

$$H(z) = \frac{1}{z^2 + z - 6}$$

(a) Find a difference equation with input $x[n]$ and output $y[n]$ for this system.

(b) What is the z-transform of $h[n+2]$?

(c) Using long division, find $h[n]$ for $n = 0, 1, 2, 3$.

(d) Find $s[n]$, the step response for this system.

TABLE 6.1 *Properties of the z Transform*

Property	*Time domain*	*Transform*	*ROC*
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$R' \supset R_1 \cap R_2$
Time shift	$x[n - n_0]$	$z^{-n_0}X(z)$	$R' \supset R \cap 0 < \|z\| < \infty$
Modulation	$z_0^n x[n]$	$X(z/z_0)$	$R' = \|z_0\| R$
	$e^{j\Omega_0 n}x[n]$	$X(ze^{-j\Omega_0})$	$R' = R$
Time reversal	$x[-n]$	$X(1/z)$	$R' = 1/R$
Differentiation	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	$R' = R$
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$R' \supset R_1 \cap R_2$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$X(z)\dfrac{1}{1 - z^{-1}}$	$R' \supset R \cap \|z\| > 1$

TABLE 6.2 *Common z Transforms*

Signal	*Time domain*	*Transform*	*ROC*
Impulse	$\delta[n]$	1	All z
	$\delta[n - n_0],\ n_0 > 0$	z^{-n_0}	$\|z\| > 0$
	$\delta[n + n_0],\ n_0 \geq 0$	z^{n_0}	$\|z\| < \infty$
Unit step	$u[n]$	$\dfrac{1}{1 - z^{-1}}$	$\|z\| > 1$
	$-u[-n - 1]$	$\dfrac{1}{1 - z^{-1}}$	$\|z\| < 1$
Exponential	$a^n u[n]$	$\dfrac{1}{1 - az^{-1}}$	$\|z\| > \|a\|$
	$-a^n u[-n - 1]$	$\dfrac{1}{1 - az^{-1}}$	$\|z\| < \|a\|$
Weighted exponential	$(n + 1)a^n u[n]$	$\dfrac{1}{(1 - az^{-1})^2}$	$\|z\| > \|a\|$
Causal sine	$(\sin \Omega_0 n)u[n]$	$\dfrac{(\sin \Omega_0)z^{-1}}{1 - 2(\cos \Omega_0)z^{-1} + z^{-2}}$	$\|z\| > 1$
Causal cosine	$(\cos \Omega_0 n)u[n]$	$\dfrac{1 - (\cos \Omega_0)z^{-1}}{1 - 2(\cos \Omega_0)z^{-1} + z^{-2}}$	$\|z\| > 1$
Damped sine	$r^n(\sin \Omega_0 n)u[n]$	$\dfrac{r(\sin \Omega_0)z^{-1}}{1 - 2r(\cos \Omega_0)z^{-1} + r^2z^{-2}}$	$\|z\| > r$
Damped cosine	$r^n(\cos \Omega_0 n)u[n]$	$\dfrac{1 - r(\cos \Omega_0)z^{-1}}{1 - 2r(\cos \Omega_0)z^{-1} + r^2z^{-2}}$	$\|z\| > r$