GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Problem Set No. 5

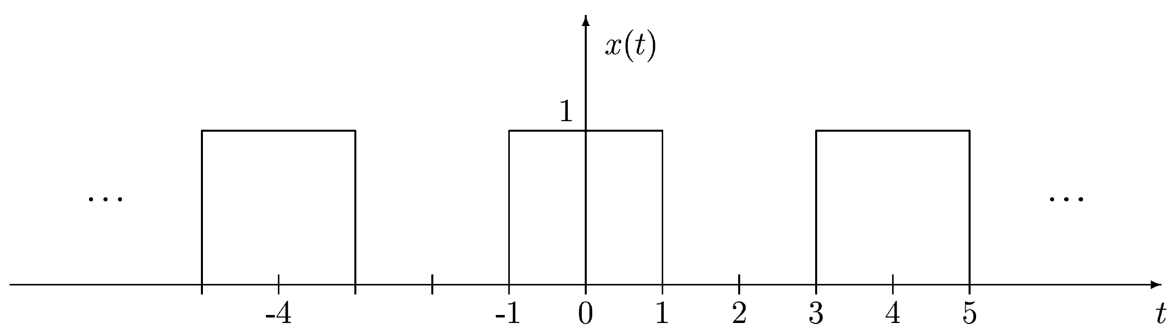
Date Assigned: February 4, 1998
Date Due: February 9, 1998

Reading Assignment: In Oppenheim and Willsky, read pp. 423-430, 439-447, 514-534, 582-607.

Homework Assignment: Turn in for grading only the starred problems: 5.1*, 5.2*, 5.4*, and 5.5*.

Practice Problems:
For practice try Problems 6.5, 7.1, 7.4, and 8.4 in Oppenheim and Willsky. These problems have answers in the back of the book.

Problem 5.1*:
Suppose that the input $x(t)$ is the square wave depicted below:



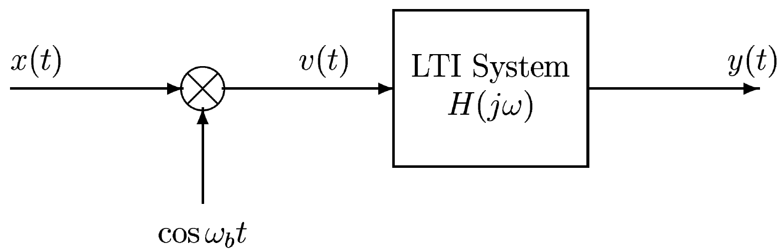
Is it *(possible or impossible)* that the corresponding output could be the signal

$$y(t) = 2\cos(2\pi t/4)$$

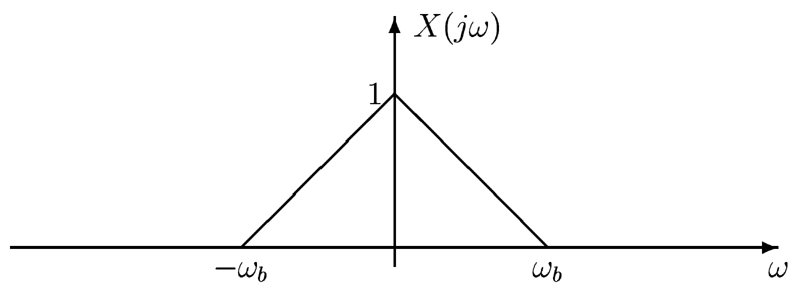
If it is impossible, state why. If it is possible, determine the frequency response $H(j\omega)$ of an ideal frequency-selective filter (lowpass, highpass, or bandpass) such that the output is as specified; i.e., determine the gain and cutoff frequencie(s) for the filter. *(A carefully labeled sketch is sufficient.)*

Problem 5.2*

Consider the following modulation system:



Assume that the input signal $x(t)$ has a bandlimited Fourier transform as depicted below



and the linear system has frequency response

$$H(j\omega) = \begin{cases} 2 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

(a) Plot the Fourier transform $Y(j\omega)$ of the corresponding output signal $y(t)$. *Note that the negative frequency portion of the Fourier transform is shaded. Mark the corresponding region or regions in your plot of $Y(j\omega)$.*

(b) Describe in words what has happened to the spectrum of the input.

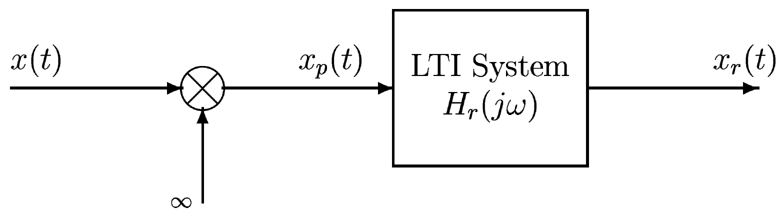
(c) Give an equation for $Y(j\omega)$ in terms of $X(j\omega)$ that holds for $\omega > 0$ and that is valid for any signal such that $X(j\omega) = 0$ for $|\omega| > \omega_b$.

(d) How could $x(t)$ be recovered from $y(t)$?

Problem 5.3
Work Problem 6.33 in Oppenheim and Willsky.

Problem 5.4*
Work Problem 8.22 in Oppenheim and Willsky.

Problem 5.5*



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

The input signal for the above sampling/reconstruction system is

$$x(t) = \cos(200\pi t + \pi/3) \qquad -\infty < t < \infty$$

and the frequency response of the lowpass reconstruction filter is

$$H_r(j\omega) = \begin{cases} T & |\omega| < \pi/T \\ 0 & |\omega| > \pi/T \end{cases}$$

where T is the sampling period.

(a) Sketch the Fourier transform $X_p(j\omega)$ for $-2\pi/T < \omega < 2\pi/T$ for the case where $2\pi/T > 400\pi$. Carefully label your sketch to receive full credit.

(b) Now assume that you can choose the sampling frequency $\omega_s = 2\pi/T$ to be any value. How would you choose ω_s so that

$$x_r(t) = A \qquad -\infty < t < \infty$$

where A is a constant? *Determine the value of A.*