GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Problem Set No. 3

Date Assigned: January 23, 1998
Date Due: January 26, 1998

Reading Assignment: In Oppenheim and Willsky, read pp. 1 231-244 and read all of Chapter 4.

Homework Assignment: Turn in for grading only the starred problems: 3.2* and 3.3*.

Problem 3.1:
Work Problem 3.35 in Oppenheim and Willsky.

Problem 3.2*:
Consider the periodic signal $x(t)$, which is defined over one period by

$$x(t) = \begin{cases} 1 & 0 < t < 2 \\ 0 & 2 < t < 4 \end{cases}$$

The period of the signal is $T = 4$.

(a) The signal $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k. Sketch the spectrum of the signal as a function of ω.

(b) The frequency response of a LTI system is

$$H(j\omega) = \begin{cases} 1 & 5\pi/4 < |\omega| < 7\pi/4 \\ 0 & \text{otherwise} \end{cases}$$

Plot the frequency response $H(j\omega)$ on the same graph as your spectrum plot.

(c) Determine the output of the system for the given input $x(t)$. Give the simplest possible equation for your answer.

Problem 3.3*:

Consider the following periodic signal, which is the input to a LTI system:

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n4)$$

(a) The input $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k.

(b) The impulse response of the LTI system is

$$h(t) = e^{-\alpha t} u(t)$$

Use convolution to obtain an equation for the output $y(t)$ when the input is the signal in part (a). *Hint: Use superposition and time invariance to find the output due each impulse.* Make a sketch of the output signal as a function of time for the case $\alpha = 2$.

(c) Determine the frequency response of the LTI system. Sketch $|H(j\omega)|$ as a function of ω. How does the shape of the frequency response depend on α?

(d) Use the frequency response and the Fourier series result of part (a) to determine a Fourier series expression for the output of the system for the given input $x(t)$. How would you choose α if you wanted the output to be essentially equal to a constant?