GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Problem Set No. 3

Date Assigned: January 23, 1998
Date Due: January 26, 1998

Reading Assignment: In Oppenheim and Willsky, read pp. 1 231-244 and read all of Chapter 4.

Homework Assignment: Turn in for grading only the starred problems: 3.2*, 3.4*, and 3.6*.

Problem 3.1:
Work Problem 3.35 in Oppenheim and Willsky.

Problem 3.2*:
Consider the periodic signal $x(t)$, which is defined over one period by

$$x(t) = \begin{cases} 1 & -2 < t < 0 \\ 0 & 0 < t < 2 \end{cases}$$

The period of the signal is $T = 4$.

(a) The signal $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k. Sketch the spectrum of the input signal as a function of ω.

(b) The frequency response of a LTI highpass filter is

$$H(j\omega) = \begin{cases} 0 & |\omega| < 3\pi/4 \\ e^{-j\omega 2} & 3\pi/4 < |\omega| \end{cases}$$

Plot the magnitude of the frequency response, $|H(j\omega)|$ on the same graph as your spectrum plot. What is the effect of the factor $e^{-j\omega 2}$ on the output waveform?

(c) Determine the output of the system for the given input $x(t)$. Give the simplest possible equation for your answer.

Problem 3.3:

Consider the following periodic signal, which is the input to a LTI system:

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n4)$$

(a) The input $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k.

(b) The impulse response of the LTI system is

$$h(t) = e^{-\alpha t}u(t)$$

Use convolution to obtain an equation for the output $y(t)$ when the input is the signal in part (a). *Hint: Use superposition and time invariance to find the output due each impulse.* Make a sketch of the output signal as a function of time for the case $\alpha = 2$.

(c) Determine the frequency response of the LTI system. Sketch $|H(j\omega)|$ as a function of ω. How does the shape of the frequency response depend on α?

(d) Use the frequency response and the Fourier series result of part (a) to determine a Fourier series expression for the output of the system for the given input $x(t)$. How would you choose α if you wanted the output to be essentially equal to a constant?

Problem 3.4*:

Consider an LTI system with

$$H(j\omega) = \frac{j\omega}{j\omega + 1} e^{-j\omega}.$$

(a) Use the tables on pp. 328-329 to find the impulse response $h(t)$ of this system.

(b) For a particular input $x(t)$, the output of the system is

$$y(t) = \frac{3}{2} e^{-3(t-2)} u(t-2) - \frac{1}{2} e^{-(t-2)} u(t-2).$$

Use the tables on pp. 328-329 to determine $x(t)$.

Practice Problems:

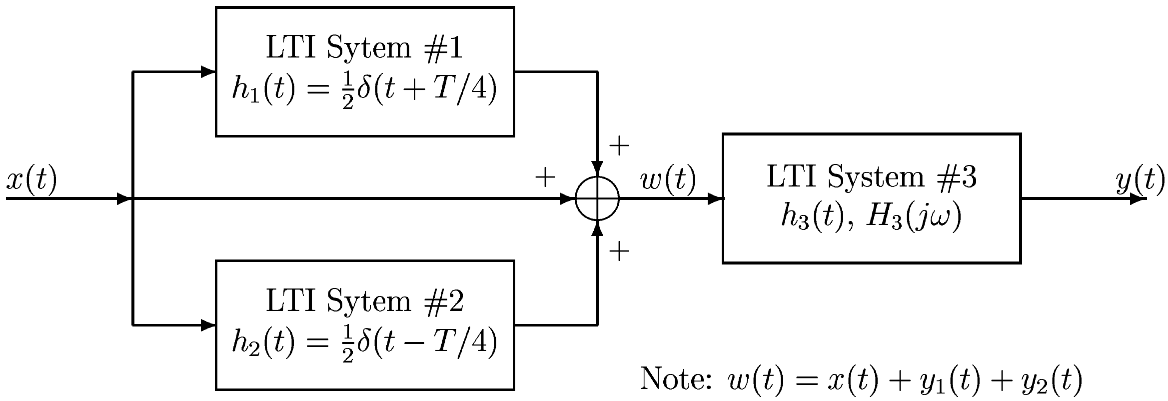
For practice try Problems 4.1, 4.2, 4.3, 4.4 and 4.6 in Oppenheim and Willsky. These problems have answers in the back of the book.

Problem 3.5:

Work Problem 4.22.(a), (b) and (c) in O & W.

Problem 3.6*:

The following system is a LTI system.



The frequency response of LTI System #3 is: $H_3(j\omega) = \begin{cases} T/4 & |\omega| < 4\pi/T \\ 0 & |\omega| > 4\pi/T \end{cases}$

(a) Determine the impulse response, $h_3(t)$, of LTI System #3.

(b) First, give an expression in terms of $h_3(t)$ for the impulse response $h(t)$ of the overall system. Then use your result from part (a) to find an equation for the overall impulse response $h(t)$. Sketch your answer showing the value of $h(0)$ and the times at which $h(t) = 0$.

(c) Determine the frequency response, $H(j\omega)$, for the overall system. Express your answer in terms of $H_3(j\omega)$ and manipulate it into a simple form so that you can easily plot it below. **Plot it.**

(d) Is the overall system stable? Is it causal? If not causal, what system could be cascaded with the given system to make an overall system that is causal?