GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Problem Set No. 2

Date Assigned: April 10, 1998
Date Due: April 17, 1998

Reading Assignment: In Oppenheim and Willsky, read pp. 182-195 and 231-239.

Homework Assignment: In all problems, write some explanation of your approach to the solution, i.e., give more than the answer. Turn in for grading only the starred problems: 2.1*, 2.2*, 2.3*, 2.4*, 2.5*, and 2.6*.

Problems with answers:

Take a look at Problem Set 2 of last quarter. These problems and solutions cover the same material as this problem set.

Problem 2.1*:

Express each of the following in a simpler form:

(a) $[\sin(4\pi t) + \cos(2\pi t)][\delta(t) + \delta(t+1)] =$

(b) $\int_{-\infty}^{\infty} \cos(2\pi\tau + \pi/3)\delta(\tau + 4)d\tau =$

(c) $\int_{-\infty}^{\infty} \cos(2\pi\tau + \pi/3)\delta(t - \tau)d\tau =$

(d) $u_1(t-3) * \cos(2\pi t + \pi/3) =$

(e) $[\delta(t) - \delta(t+1)] * [\delta(t-1) + \delta(t-2)] =$

(f) $u(t) * \delta(t-3) * u_1(t) =$

Problem 2.2*

An LTI system has impulse response $h(t) = u(t) - u(t-T)$.

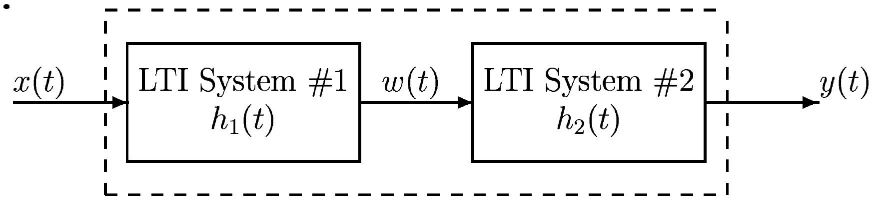
(a) Find the system function $H(s)$ of this system. What is the region of convergence?

(b) Find the frequency response $H(j\omega)$ of this system.

(c) Using convolution, find the output of this system for the input $x(t) = e^{j\omega t}u(t)$ for all t.

(d) After a certain time t_0, the output of the system is identical to $H(j\omega)e^{j\omega t}$. What is t_0?

Problem 2.3*:



The first system has impulse response

$$h_1(t) = e^{-\alpha t}u(t),$$

and the second system has impulse response

$$h_2(t) = \alpha\delta(t) + u_1(t),$$

where $u_1(t) = \delta^{(1)}(t)$.

(a) Determine the impulse response $h(t)$ of the overall cascade system; i.e. $h(t)$ such that $y(t) = x(t) * h(t)$.

(b) In this case, System #2 is called the *inverse* of System #1 and vice versa. Explain why.

Problem 2.4*:

A linear time-invariant system has impulse response

$$h(t) = e^{-2t}u(t)$$

and frequency response

$$H(j\omega) = \frac{1}{2 + j\omega}.$$

Use superposition to find the output due to the input

$$x(t) = 10 + 10\cos(2t) + \delta(t-3).$$

Problem 2.5*:

Consider the periodic signal $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n10)$.

(a) The input $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k.

(b) The impulse response of an LTI system is

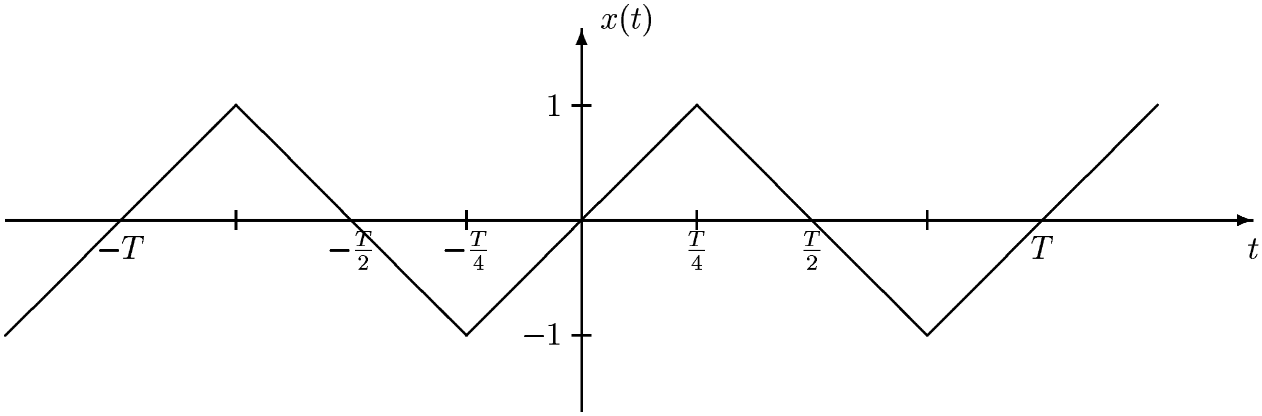
$$h(t) = u(t) - u(t-10).$$

Give an equation for $y(t)$ in terms of $h(t)$ when its input is $x(t)$ as in part (a). Make a carefully labelled plot of the corresponding output, $y(t)$.

(c) Determine the frequency response of the LTI system and use it to determine the coefficients b_k in the Fourier series for the output

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}.$$

Problem 2.6*:



(a) The periodic signal $x(t)$ plotted above is the input to an ideal differentiator system, whose output is therefore

$$y(t) = \frac{dx(t)}{dt}.$$

Find and plot carefully the output signal $y(t)$.

(b) You should have found that the output is a periodic square wave. Determine the Fourier coefficients b_k for this output signal in the Fourier series representation

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}.$$

(c) Since the input $x(t)$ is a periodic signal, it also can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t},$$

and since the differentiator is an LTI system, its output can be represented in terms of the input Fourier coefficients as

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}.$$

where $H(j\omega)$ is the frequency response of the differentiator. Use all this and the result of (b) to find the Fourier coefficients a_k of the input signal $x(t)$.